Possibilistic Physics

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Many aspects of the world are non-deterministic. The concepts and methods of probability theory and quantitative statistics have entered, for example, the realms of social sciences, biology, and finance. All of these areas are non-fundamental descriptions of some aspects of our world where the appearance of non-determinism and probability is an emergent phenomenon and originates from averaging over unknown parameters. However, since the advent of quantum theory in the early 20th century, even fundamental physics as the most basic description of nature has become probabilistic. In a fundamental theory of nature, the appearance of probabilistic features cannot be emergent, since by the very definition of fundamental physics there cannot exist any unknown and more fundamental parameters that need to be averaged over.

This raises the question whether physics actually obeys the laws of probability theory, on the most fundamental level. If yes, from which physical principle could the laws of probability—or its quantum counterparts—be derived? If no, what are possible alternatives to probability? It is this second question I am going to ponder here; since my personal belief is that no assumption is too elemenatary for careful scrutiny. I want to suggest that one can possibly completely despense of probabilities and replace them by the concept of *possibility*: a physical theory then would just state which events are possible, which are impossible, and nothing else.

Therefore, I do not have an answer to the question,

"What's ultimately possible in physics?"

Instead, I want to argue that besides trying to find answers to this question within the framework of a given physical theory, one can also turn this procedure upside down and regard the answers to this question as *defining* a physical theory.

Specifying a physical theory by saying what is possible to occur and what is impossible to occur could be called *possibilistic physics*. In this essay, I will describe a very general mathematical framework for possibilistic physics. As a start, I want to focus on the possibilistic analogue of general probabilistic theories, and in particular two-party Bell tests. Recall that a Bell test is a system which allows for detection of quantum non-locality, one of the most fascinating facets of quantum mechanics. Its possibilistic analogue will be described and characterized. Also, it is found that Spekkens' toy theory of quantum mechanics [Sp] is inconsistent in the usual probabilistic interpretation, but is a perfectly fine example of a possibilistic theory.

I am well aware that some of the philosophical arguments discussed in the last part of this essay are not totally imperturbable. They should rather be regarded as a study of feasibility: how much can one possibly to do with the concept of possibility and which arguments speak in favor of using it? In particular, how does this concept compare to probability? Further investigations of both the mathematical and the philosophical aspects are necessary.

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Recap of general probabilistic theories

Since possibilistic theories will be defined in analogy with general probabilistic theories, we should start by shortly recalling the latter before delving into the former. Note that my use of "prediction" and "theory" is non-standard terminology.

Quantum physics is, both in theory and in experiment, fundamentally non-deterministic. In an experiment, the outcome of a measurement cannot be predicted, even when the initial state of the system is known completely. Instead, what can be inferred from the theory is that the outcomes of many repetitions of the same experiment sould be independent random variables identically distributed according to a certain probability distribution. This implies that the relative asymptotic frequency of each outcome will converge to a certain value in the unit interval $[0, 1]$. It is this value that can be compared between theory and experiment.

Predictions and theories. A convenient framework for probabilistic theories like quantum mechanics is given by the concept of *general probabilistic theory.* To begin explaining this term, think of a physical system, which is to be observed, together with a finite number of measurements that can be performed on this system. Then a *prediction* is defined to be a specification of a probability distribution over the outcomes of each measurement. More precisely, for a set of n different measurements, where the ith measurement has the set $\mathcal{O}_i$ as its set of possible outcomes, a prediction is given by numerical values in terms of probability assignments

$$P_i : \mathcal{O}_i \longrightarrow [0, 1], \qquad \forall\, i = 1, \ldots, n\,,$$

such that for each measurement, some outcome occurs with certainty:

$$\sum_{x \in \mathcal{O}_i} P_i(x) = 1, \qquad \forall\, i = 1, \ldots, n\,. \tag{1}$$

In this essay, the term "model" will often be used as a synonym for "prediction".

In quantum theory, and certainly in any other sensible theory of physics, the prediction depends on the initial state of the system and on the interaction dynamics between the system and the measuring apparatus. Therefore, no sensible theory would only allow for a single prediction. Instead, a general probabilistic theory is defined to be a convex subset of the set of all predictions: those that are allowed by the theory. Convexity is required since one can always take stochastic combinations of initial states to obtain the corresponding convex combination of predictions as the total prediction—e.g. flip a coin that decides which initial state to use in the experiment. We will later see that this is violated in the probabilistic interpretation of Spekkens' toy theory, turning it inconsistent.

This ends the definitions of prediction and (general) probabilistic theory. Note that a prediction makes no statement about what happens with the system after the measurement, and a probablistic theory makes no statement about how the prediction depends on the initial state. In this sense, the formalism only captures a tiny aspect of physics.

The Bell test system. As an example system, let us analyze a Bell test experiment. This system consists of two observers, commonly called Alice and Bob, in spacelike separated regions. The simplest situation—not

		a	
		$+$	$-$
b	$+$	0.20	0.31
	$-$	0.39	0.10

(a) When Alice measures a and Bob measures b.

		a		a'	
		$+$	$-$	$+$	$-$
b	$+$	0.20	0.31	0.27	0.03
	$-$	0.39	0.10	0.37	0.33
b'	$+$	0.57	0.20	0.39	0.00
	$-$	0.02	0.21	0.25	0.36

(b) For the full Bell test, where Alice and Bob each can choose among two measurements.

Figure 1: Example predictions in a general probabilistic theory.

yet the Bell test—is that Alice measures a and Bob measures b, measurements that both have two possible outcomes, say, $+$ and $-$. In order to be able to talk about correlations between a and b, it is necessary to consider a and b together as a single measurement with four possible outcomes

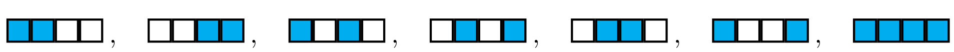
$$\mathcal{O}_{a,b} = \big\{(+,+),\\ (+,-),\\ (-,+),\\ (-,-)\big\}. \tag{2}$$

A sample prediction for this measurement is shown in figure 1(a). A slightly more complicated system is the *Bell test*, where Alice can choose between the measurements a and a', whereas Bob can choose between the measurements b and b'. Again, the set of possible outcomes for each measurement is $\{+,-\}$. In total, this gives the four measurement combinations (a,b), (a,b'), (a',b), and (a',b'), which ought to be regarded as the elementary measurements. These measurements all have the same set of outcomes (2). A sample prediction is shown in figure 1(b); note that the probabilities sum to 1 in each of the four subsquares. Since the marginals for Alice do not depend on the choice of Bob's measurement, this sample prediction belongs to the theory "no signalling from Bob to Alice". And since the marginals of Bob do depend on the Alice's choice of measurement, this sample prediction does not belong to the theory "no signalling from Alice to Bob".

Spekkens' toy theory and general possibilistic theories

On Spekkens' toy theory. I will now show that Spekkens' toy theory of quantum mechanics [Sp] does not fit into the framework of general probabilistic theories. This will give a motivation for the introduction of the framework of *general possibilistic theories*, of which Spekkens' toy qubits are an example.

Recall that the possible pure states of a Spekkens qubit are exactly the following:



As noticed in [Sp, III.A], these states are somewhat analogous to spin states of a spin 1/2 particle for which the spin is aligned along one of the coordinate axes. One incarnation of this correspondence is that the (reproducible) measurements [Sp, III.D] that can be performed on a toy qubit are, up to relabelling the outcomes like $+ \leftrightarrow -$, represented as in the table

	■■□□	□□■■	■□■□	□■□■	□■■□	■□□■	■■■■
⊞⊞⊟⊟	$+$	$-$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
⊞⊟⊞⊟	$\pm$	$\pm$	$+$	$-$	$\pm$	$\pm$	$\pm$
⊞⊟⊟⊞	$\pm$	$\pm$	$\pm$	$\pm$	$-$	$+$	$\pm$

(3)

Each column of this table specifies the prediction in that state. The entries $+$ and $-$ denote definite outcomes where the measurement yields that outcome with certainty. Spekkens [Sp, D] writes about the "$\pm$" entries standing for indefinite outcomes,

> [...] then the outcome is not determined. In a large ensemble of such experiments, one expects the two outcomes to occur with equal frequency.

Therefore, in the terminology of general probabilistic theories, the Spekkens qubits define a theory with exactly seven possible predictions. The problem now is that these seven predictions do not form a convex subset of the space of all probability assignments, turning outcomes of experiments ambiguous. For example, suppose we set up the following experiment: use a random initialization of the qubit such that it is initialized in the state ■■□□ with a probability of $2/3$, and in the state □□■■ with a probability of $1/3$. Then upon conducting the ⊞⊞⊟⊟ measurement, the rules of probability dictate that $+$ should occur with an asymptotic frequency of $2/3$. On the other hand, the random initial state of the system should also be describable in terms of a mixed state; especially so since Spekkens' states are supposed to be states of subjective knowledge, not of objective existence. But the only (non-pure) mixed state in his theory is ■■■■, which yields, according to the rule quoted above, the outcome $+$ with a probability of merely $1/2$.

More generally, such an argument makes it clear that—due to a violation of the convexity condition—no general probabilistic theory can have only a finite number of predictions. To avoid this problem, one could certainly take the convex hull of all predictions defined by the six pure states, and regard the resulting region as a modified Spekkens theory. The knowledge balance principle advocated in [Sp] would still be satisfied. However, this would contravene the combinatorial flavor of the theory as having only a finite number of states.

The alternative is to leave the paradigm of general probabilistic theories. If one regards a "$\pm$" in (3) as merely stating that "both outcomes are possible", the problem about random mixtures disappears: in both ways of reasoning, one obtains nothing more but the statement "both outcomes are possible". Actually, it shouldn't really be much of a surprise that a theory of physics which is discrete and combinatorial in flavor can only make predictions of a nature that are themselves discrete, combinatorial, and therefore non-quantitative!

This is the approach that will be taken here. I will now start to outline a general framework for possibilistic theories of physics, which is analogous to the framework for general probabilistic theories outlined in the previous section. Spekkens' toy qubits serve as a perfectly generic example.

Possibilities and possibilistic predictions. In contrast to ordinary probability theory, where every event gets assigned a probability value lying in $[0,1]$, we now merely assign to every event a possibility value. A possibility value is an element of the set $\{0,*\}$. The interpretation is that an event with possibility value 0 can be excluded and will certainly not occur, whereas an event with a possibility value of $*$ can occur, but it does not have to. Mathematically, this kind of possibility theory can be viewed as probability theory with coefficients in the semiring $\{0,*\}$, where the algebraic operations on this semiring are given by[1]

$$\begin{aligned} 0+0&=0, \qquad 0+*=*, \qquad *+*=* \\ 0\cdot 0&=0, \qquad 0\cdot *=0, \qquad *\cdot *=*. \end{aligned} \tag{4}$$

[1] Formally, this is isomorphic to the semiring of boolean truth values $(\top,\bot,\vee,\wedge)$.

In a physical setting, consider again n measurements where the set of outcomes of the ith measurement is $\mathcal{O}_i$. Now, a *possibilistic prediction* is specified in analogy with the probabilistc case by a possibility distribution, which is a function

$$\pi_i : \mathcal{O}_i \longrightarrow \{0, *\}, \qquad i = 1, \ldots, n$$

such that for each measurement, at least one outcome is possible. By virtue of the algebra (4), this condition is totally analogous to (1),

$$\sum_{x \in \mathcal{O}_i} \pi_i(x) = *, \qquad i = 1, \ldots, n \tag{5}$$

As a first example, consider a single binary measurement with $\{+, -\}$ as its set of possible outcomes. Then there are three possible predictions:

$$\begin{array}{lll} & \pi(+) = *, \quad \pi(-) = 0. & \text{Here, the outcome is } + \text{ with certainty.} \\ \text{or} & \pi(+) = 0, \quad \pi(-) = *. & \text{Here, the outcome is } - \text{ with certainty.} \\ \text{or} & \pi(+) = *, \quad \pi(-) = *. & \text{Here, both outcomes can occur.} \end{array}$$

For the third prediction, both outcomes are possible, and this is the only statement made by this prediction. Nothing at all is implied about how the two outcomes relate to each other.

Possibilistic theories. Just as in the probabilistic case, a *possibilistic theory* consists of a subset of all predictions. Like a general probabilistic theory needs to be closed under convex combinations corresponding to a probabilistic random choice of initial state, a possibilistic theory needs to be closed under sums (4), corresponding to a possibilistic choice of initial state. For suppose that we have two initial states available, corresponding to predictions π and π'. Now we let a coin flip[2] decide which initial state to use. In the combined experiment, an outcome will be possible if and only if it is possible in π or in π'. Therefore, the prediction for the combined experiment is $\pi + \pi'$.

The possibilistic Bell test. As a relevant example system, I now want to give sample predictions for a possibilistic Bell test. Figure 2 is a self-explaining analogue of figure 1. Note that each of the four subsquares contains at least one $*$, so that (5) holds. Now, the possibilistic marginals of Alice do not depend on the choice of Bob's measurement, and hence this sample prediction belongs to the possibilistic theory "no signalling from Bob to Alice", which are those satisfying the relation

$$\pi_{a,b}(x,+) + \pi_{a,b}(x,-) = \pi_{a,b'}(x,+) + \pi_{a,b'}(x,-)$$

together with its obvious variants. But when Bob chooses to measure b', he gets that the outcome $+$ is impossible when Alice measures a, whereas the outcome $+$ is possible when Alice measures a'. Hence, the prediction in figure 2(b) does not belong to the possibilistic theory "no signalling from Alice to Bob".

The search for possibilistic quantum mechanics. Quantum mechanical theories are a very special class of general probailistic theories. One of the original hopes of this work was that there might be a combinatorial framework for a possibilistic analogue of quantum mechanics encompassing the Spekkens model and also comprising other models that have interesting combinatorial properties. So far, this hope has remained unfulfilled. I will briefly digress to describe the ideas involved. The crucial structure of quantum theory is that of a Hilbert space, i.e. a vector space together with a positive definite bilinear form. The combinatorial

[2] Here, it doesn't matter whether the coin flip itself is probabilistic or possibilistic.

		a +	a −
b	+	0	*
	−	0	*

(a) When Alice measures a and Bob measures b.

		a +	a −	a' +	a' −
b	+	0	*	0	*
	−	0	*	*	0
b'	+	0	*	*	0
	−	0	0	*	*

(b) For the full Bell test, where Alice and Bob each can choose among two measurements.

Figure 2: Example predictions in a general possibilistic theory.

analogue of a vector space is given by the mathematical structure of a *matroid* (see e.g. [Ox]). In the possibilistic framework, numerical values of the scalar product are irrelevant; the only information retained is whether a scalar product vanishes or not, i.e. the orthogonality relation. This led me to the study of matroids having a compatible orthogonality relation. With a good definition of compatibility, the linear subspaces of the matroid form an orthomodular lattice, hence this is closely connected to quantum logic. Then the whole structure of matroid plus orthogonality can also be encoded in a single function "orthogonal complement" going from subsets to subsets, such that the matroid linear hull of a subset can be recovered as the orthogonal bicomplement. But then since the orthogonality relation itself determines the orthogonal complement, the matroid structure is determined by the orthogonality relation alone. Such a relation is most conveniently represented as a graph, and therefore one can also speak of an *orthogonality graph*. Not every graph is an orthogonality graph in this sense; in fact, all the examples of orthogonality graphs that I could find can be decomposed as an orthogonal sum into one- and two-dimensional components, so that no interesting examples are known to me. In particular, the Spekkens two-qubit theory [Sp, IV] does not fit into such a framework.

There are two further directions that should be explored along these lines: first, to see if categorical quantum mechanics [AC] contains possibilistic theories. And second, to draw the comparison to modal logic and its "possible worlds".

Possibilistic Bell inequalities

The possibilistic Bell test scenario has been mentioned previously. In this section, I want to formulate the theory of possibilistic local hidden variables, mention the computation of possibilistic Bell inequalities, and show that possibilistic Popescu-Rohrlich boxes violate these inequalities.

The theory of possibilistic local hidden variables. To begin, a deterministic local hidden variable model is defined to be a prediction where each measurement has a definite outcome, and Alice's and Bob's measurements are independent in the sense that a composite outcome (x, y) is possible if and only if x is possible for Alice's measurement and y is possible for Bob's measurement.

Then, a possibilistic local hidden variable model is defined to be a combination of deterministic local hidden variable models. This means that in can be written in the form

$$\pi_{a,b}(x,y) = \sum_{\lambda} \pi_a^{\lambda}(x) \cdot \pi_b^{\lambda}(y) \tag{6}$$

		a		a'	
		$+$	$-$	$+$	$-$
b	$+$	$*_{1,2}$	0	$*_1$	$*_2$
	$-$	0	$*_{3,4}$	$*_3$	$*_4$
b'	$+$	$*_1$	$*_3$	$*_{1,3}$	0
	$-$	$*_2$	$*_4$	0	$*_{2,4}$

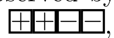
(a) A possibilistic hidden variable model occuring in the Spekkens theory.

		a		a'	
		$+$	$-$	$+$	$-$
b	$+$	$*$	0	$*$	0
	$-$	0	$*$	0	$*$
b'	$+$	$*$	0	0	$*$
	$-$	0	$*$	$*$	0

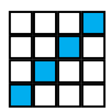
(b) A possibilistic Popescu-Rohrlich box.

Figure 3: More example predictions for the possibilistic Bell test system.

and analogously for $a \leftrightarrow a'$ and $b \leftrightarrow b'$. The parameter λ indexes the possibility distributions π_a^λ, $\pi_{a'}^\lambda$, π_b^λ and $\pi_{b'}^\lambda$ for Alice's and Bob's subsystems separately. The sum over λ allows for classical random-possibilistic correlations between the systems. Equation (6) corresponds to the representation of a probabilistic local hidden variable model as an integral over deterministic product models.

By checking all the possible combinations of measurements that can be done, one can show that the entangled states in the two-qubit Spekkens model are indeed always local hidden variable models. For the (inconsistent) probabilistic interpretation, this also has been observed by Spekkens [Sp, VII]. Figure 3(a) shows the possibilistic prediction obtained by measuring $a = b =$ , $a' = b' =$  in the state

$$(1 \cdot 1) \vee (2 \cdot 2) \vee (3 \cdot 3) \vee (4 \cdot 4) =$$



The indices on the $*$'s in figure 3(a) indicate a decomposition into deterministic local hidden variable models.

Popescu-Rohrlich boxes. Not all no-signalling predictions have local hidden variable models. In particular, the prediction shown in figure 3(b) does not; the pattern of 0's and $*$'s is reminiscient of a probabilistic Popescu-Rohrlich box, and therefore one might regard it as a possibilistic Popescu-Rohrlich box. By applying the permutations $+ \leftrightarrow -$, $a \leftrightarrow a'$ and $b \leftrightarrow b'$, one obtains eight different possibilistic Popescu-Rohrlich boxes.

Bell inequalities. How can one recognize a hidden variable model when one sees one? In the probabilistic case, there is a set of necessary and sufficient criteria: the CHSH inequalities. In the present framework, an analogous characterization turns out to be possible. Using an appropriate variant of Fourier-Motzkin elimination, it can be proven that any subset of predictions that is closed under the possibilistic sum (4) can be characterized by a collection of inequalities which are linear with respect to the operations (4). In fact, this method can also be used to *calculate* these inequalities. Besides the no-signalling equations and the requirement that each of the four subsquares needs to contain at least one $*$, one ends up with the non-trivial Bell inequality

$$\pi(L) \leq \pi(R_1) + \pi(R_2) + \pi(R_3) \tag{7}$$

where the arguments represent measurement outcomes as depicted in figure 4, together with the obvious permutations $+ \leftrightarrow -$, $a \leftrightarrow a'$ and $b \leftrightarrow b'$ of these. Due to the form of the algebraic operations (4), such an inequality (7) is equivalent to the following implication:

$$\text{If } \pi(L) = *, \text{ then also } \pi(R_i) = * \text{ for at least one } i \in \{1, 2, 3\}. \tag{8}$$

		a		a'	
		$+$	$-$	$+$	$-$
b	$+$	L		R_1	
	$-$				
b'	$+$	R_2			
	$-$				R_3

Figure 4: Up to permutations, this represents the only possibilistic Bell inequality (7).

It is immediate to check that any deterministic hidden variable model satisfies the implication (8). But then by linearity, any hidden variable model satisfies this inequality. On the other hand, the Popescu-Rohrlich box from figure 3(b) violates several of the permutations of this inequality. It is guaranteed by the mathematics underlying the computation that any prediction that is not a hidden-variable model will violate at least one of (7)'s permutations $+ \leftrightarrow -$, $a \leftrightarrow a'$ and $b \leftrightarrow b'$.

Further details on the possibilistic Bell inequalities and the Fourier-Motzkin-type algorithm underlying their computation will appear elsewhere. A `C` implementation of this algorithm can be downloaded from [Fr], together with the input file used for the computation of figure 4.

Discussion of probabilistic vs. possibilistic

Obviously, every probabilistic prediction determines a corresponding possibilistic one, simply by taking an outcome to be impossible if it has probability 0, and taking it to be possible otherwise. From this naive point of view, probabilistic theories have a higher predictive power than their possibilistic counterparts, since an actual numerical value for a probability is certainly better than just the statement "the outcome can occur". This is a possible argument against possibilistic physics. In this section, I want to present some arguments in favor of it.

Hypothesis testing is possibilistic. How does one compare a probabilistic theory with experiment? In accordance with the scientific method, the goal of an experiment is to try and falsify the theory. In most cases—due to the non-deterministic nature of probability—this cannot be done in a single measurement alone. Instead, a series of measurements is required.

For simplicity, I want to consider tests of a single hypothesis only and not comparison tests of two or more hypotheses, as they are usually done in statistics. So consider a coin flip as an experiment having the outcome h (heads) or t (tails). For example, the hypothesis may be that the coin is fair, i.e. that each outcome occurs with probability 1/2. Our experimentalist friend would run the experiment a certain number of times, say N times, and naively he would compare the number of heads N_h with the numbers of tails $N - N_h$. He would then run a statistical test on the pair (N_h, N) to determine how statistically significant the deviation from the expected value $(N/2, N)$ turns out to be. Based on an initially stipulated significance level, the hypothesis will be regarded either as confirmed or as falsified. See figure 5 for a schematic illustration: the fair coin hypothesis gets accepted in the square-filled region and rejected in the grey ruled region of (N_h, N)-space.

What this means is nothing but that the theory predicts the outcomes in the square-filled region to be possible, and the outcomes in the grey ruled region to be impossible. In this sense, hypothesis testing is possibilistic. Of course this is slightly misleading since the boundary between the acceptance region and the

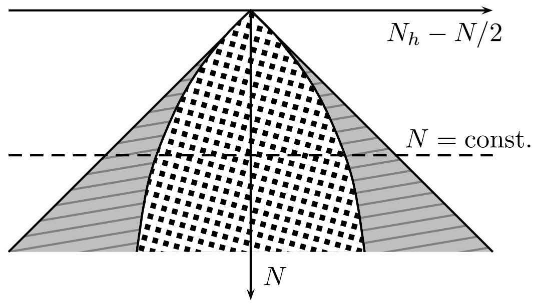


Figure 5: Regions of acceptance and rejection for a statistical significance test of the hypothesis "The coin is fair" (schematic).

rejection region is not sharp. But then in fundamental physics, how do we know that this is the way it has to be? Upon regarding the hypothesis test as part of the physical theory itself, one can set up a possibilistic theory of measurement series and thereby obtain a precise sharp boundary between the acceptance region and the rejection region. Then given the theory, no debate over the statistical significance of experimental deviations from theoretical predictions would be possible. It is in this sense that a possibilistic theory—on the level of *series* of measurements—has a *higher predictive power* than a probabilistic one: the conditions for its falsification are absolutely clear-cut.

Probabilistic theories specify possible probabilities. There is another sense in which a probabilistic theory has a possibilistic aspect: any prediction is either allowed by the theory, or it is not. For example in the Bell test system, the boundary between the quantum region and the non-quantum region is absolutely sharp. The outcome probabilities within the quantum region can possibly occur in our quantum world, while the others cannot. So why is it that in this case, one uses probability on one level, but possibility on the other level?

What is the physics behind randomness? There is another important issue pertaining to series of measurements, which can also be illustrated using the fair coin example. Suppose that our experimentalist friend has conducted the experiment 40 times, and he has obtained the following series of outcomes,

`hhhhhhhhhhhhhhhhhhhhtttttttttttttttttttt`.

Then although the relative frequency of the two outcomes is exactly as expected, no sensible physicist would accept this series of outcomes as a confirmation of the fair coin hypothesis. Instead, he would possibly prefer an explanation like,

> "Some evil theorist has been tinkering with the apparatus while I was on coffee break."

The same problem appears for the following conceivable series of outcomes,

`hthththththththththththththththththththt`.

The reason that both of the series of outcomes cannot be accepted as realistic is that a probabilistic theory actually predicts more than just the relative frequency of outcomes. It also predicts that the measurements ought to be statistically independent! However, actual criteria for when a series of outcomes is reasonably random are hard to come by; the mathematically soundest one is the concept of *Kolmogorov randomness*, which is based on concepts from computer science. However, Kolmogorov randomness as well as related measures of randomness are so intricate that a *direct* physical justification for the observation that realistic series of outcomes show statistical independence seems out of reach. Possibilistic physics would now come into the game if one can find a possibilistic theory of measurement series, where the possible outcomes all look suitably random, such that this theory can be derived from some basic physical principles.

Almost surely means certainly. In any probabilistic theory, an event that has probability 1 occurs with absolute certainty, while an event with probability 0 will certainly not occur. This assumption is necessary since otherwise tests of statistical significance would be meaningless. This excludes the possibility that some fixed measurement outcome occurs sometimes, but so infrequently that its occurence has an asymptotic relative frequency of 0. Why should this possibility be unrealistic? Finding an answer to this question is another problem that does not arise in possibilistic physics.

Flying saucers. Finally, let me give a possibilistic analysis of a situation described by Feynman in his *Messenger Lectures* [Fe]:

> I had a conversation about flying saucers some years ago with a layman—because I am scientific I know all about flying saucers. So I said, "I don't think there are flying saucers". So my antagonist said, "Is it impossible that there are flying saucers? Can you prove that it's impossible?" "I don't know, I can't prove it's impossible. It's just very unlikely". At that he said, "You are very unscientific. If you can't prove it impossible, then how can you say that it's unlikely?" But that is the way that is scientific. It is scientific only to say what's more likely and [sic] less likely, and not to be proving all the time possible and impossible.

Let me try to circumvent the deep waters of the interpretation of subjective probability here and only mention the possibilistic alternative to Feynman's point of view. One may consider the known answers to questions of the form

"Does this photograph show a real flying saucer?"
"Has this person been abducted by actual aliens?"

as a series of outcomes of a measurement taking values in the set $\{\text{yes}, \text{no}\}$. To the best of my knowledge, many of these questions have received a definite "no", but so far none of them has seen a definite "yes". Therefore, the prediction

$$\pi(\text{yes}) = *, \quad \pi(\text{no}) = 0$$

has already been falsified. The two remaining predictions

$$\begin{aligned} \text{Feynman:} \;\; & \pi(\text{yes}) = 0, \quad \pi(\text{no}) = * \\ \text{layman:} \;\; & \pi(\text{yes}) = *, \quad \pi'(\text{no}) = * \end{aligned}$$

are still valid hypotheses. However, since the first prediction π can be falsified while the second π' cannot, Feynman's prediction that flying saucers do not exist is clearly superior over the layman's hypothesis that both the existence and the non-existence of flying saucers is possible. More generally, suppose that the set of possible outcomes for prediction 1 is a proper subset of the set of possible outcomes for prediction 2. Then, as long as prediction 1 is consistent with observations, it should be preferred over prediction 2, due to its higher predictive power. This may be regarded as a manifestation of Occam's razor, in the sense that the more parsimonious explanation is the better one.

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