The Production of Time

Adam Helfer

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The nature of time is arguably the most fundamental question in physics today. It includes the daily mystery of reconciling the perceived unfolding of events with the apparently almost entirely reversible laws of the physics which successfully describes so much of them, the puzzle of the very existence (to good approximation) of a "cosmic time," the question of what replaces conventional notions of time at Planck scales near the origin of the Universe (and so the question of in what sense the Universe *had* an origin, and whether there is any meaning to what happened "before the origin"), as well as the ultimate end of the Universe.

We have no clear indication at present that we are close to really understanding any of these points. What I shall do here is to explore some of them from the perspectives offered by attempting to reconcile two successful and (from our present view) fundamental theories, general relativity and quantum field theory. I shall argue that there is good reason to think that the measurement process and issues related to it are bound closely with understanding the nature of time, and, indeed that measurement creates "time" in a sense which is as important as the usual relativistic understanding of time.

1 General Relativity

I think most physicists would agree that general relativity coupled to appropriate classical sources gives a very beautiful and satisfying picture of the development of systems over time, in the sense that the space–time resulting from such a description can be relied on within well-understood limits to describe physical reality, and contains within it a prescription for understanding and measuring time: insofar as general relativity is applicable, time is measured by the geometry of space–time itself, the "proper time" along causal curves.

On the other hand, the matter terms chosen to be sources for general relativity can hold within themselves physics which gives one pause about the completeness of such a classical general-relativistic picture. The most vivid of these are contained in "time bomb" scenarios: situations where the matter, at some point, because of an infinitesimal internal change, winds up altering its behavior and, in turn, setting off enormous changes in the structure of space–time itself. In this way, depending on whether the "bomb" goes off or not, one can arrange for galaxies to collide or not, for black holes to form or not, and for the entire causal structure of space–time (including not just quantitative measures of time but the relative temporal orderings of events) to the future of the "explosion," to be subject to the behavior of the "bomb."

One might take the position that such situations are self-consistently described by classical physics; they simply depend sensitively on initial conditions (the precise time the bomb will go off, the precise nature of the explosion). Yet it is not hard to arrange that these classical uncertainties are dominated by quantum ones. Then, whatever the classical model's self-consistency, it neglects essential physical effects and is not appropriate for predictive work. In a case like this, one can presumably *describe* and *measure* the evolution of the space–time and thus of measures of time within it by classical constructs, one does not have an adequate way of *predicting* it from initial data, in more than a statistical sense.

This is closely related to a discrepancy between how general relativity describes the world and how we experience it. We have a clear sense of existing in the present, with a future which has yet to be created. And while general relativity does an excellent job of describing the course of events and their causal relations once they have transpired, it does not, except for restricted forms of classical matter, give an absolute way of predicting the future.

It is no accident that these potential failures of predictability seem to depend essentially on properties of the matter terms, rather than on more purely gravitational factors. This reflects the common view that some form of "cosmic censorship" should hold and hence that in some sense the maximal Cauchy development of the space–time should be inextensible; were cosmic censorship to fail, one would have more essentially gravitational sources of unpredictability.

2 Quantum Measurement

I want to turn now to the quantum realm. I will adopt the Heisenberg picture, so that state vectors do not change (except when they are reduced by measurements — an issue I will take up soon), and the quantum operators evolve. There has, of course, been a great deal of debate about different possible interpretations of quantum theory. I would like to focus this by beginning with a physical question. (I will leave it to my readers to judge how well different interpretations address this question.)

I would suggest that the main unanswered question in quantum theory is, When does a measurement occur? This question is not directly answered by any conventional interpretation (although it has been taken up within the framework of "objective reduction" theories, cf. Leggett 2002). More precisely, under what circumstances can we say that a measurement will occur, or will be likely to occur, and what observable will be measured, or be likely to be measured?

While in a practical sense we answer this in laboratories all the time, that practical answer seems to come down to a matter of human consciousness — of reading a value. While some have, on this basis, argued that consciousness is an essential part of the reduction procedure, I find this very hard to accept. (Is it really plausible that the Universe was in a gross macroscopic superposition of states until consciousness developed? And, what started consciousness?) Setting explanations via consciousness aside, then, we have at present little to say in answer to this question.[1]

One reason that this question is hard is that there is a great deal of freedom in "placing the classical/quantum cut." To a large extent, it does not make any practical difference whether we regard an experiment as done when its results are printed out or when we look at the print-out. Within the framework of quantum theory, we may say that, as long as the observable to be measured commutes with the Hamiltonian, it makes no difference at what time it is observed.[2] Yet while this invariance is substantial, it cannot be pushed to the point of asserting that measurements do not occur.

When a measurement does occur, I shall say that, insofar as conventional quantum theory is adequate to describe what happens, the state vector reduces. I am aware that many working physicists prefer to avoid this term. On the other hand, the actual computations that are done (projecting to the relevant eigenspace of the operator measured) are agreed on by everyone, and "reduction" seems to be a good name for this. The term is not meant to include any additional interpretational baggage.

As is well-known, time is not an observable in the technical sense in quantum theory; it occurs as a parameter rather than an operator. Thus, within quantum theory itself, we have no direct theory of the measurement of time. Yet perhaps this is not the right way of looking at things. Perhaps time is not merely a parameter, but another sort of thing, in quantum theory.

3 Happening in Quantum Theory

Our knowledge and description of a quantum system thus comes from two sources, the operators (whose dynamics are ultimately thought to be governed by relativistic wave equations) and the state vector. Indeed, from the strict point of view, nothing observable ever *happens* except when a measurement is made. All of the intervening dynamics of the field variables simply serves to tell us how the operators at one time are related to those at another. Solving these dynamics allows us to predict the odds of getting different measurements of different observables at a future time, but we learn nothing definite until we make a measurement. (This is one way of viewing the "problem of time.")

[1] I should mention one point at which this question is believed to be especially important, and on which there is an approach to it in contemporary work. This is the question of when quantum fluctuations in the early Universe "freeze out" and become classical; cf. Peacock (1999). While there is in the literature a definite prescription for this in terms of "horizon crossing" in particular cosmological models, and while this prescription may well turn out to have merit, it should be emphasized that it really amounts to a new hypothesis and is not a consequence of any accepted physical theory.

[2] More precisely, the requirement is that the observable should commute with the Hamiltonian on the space in which the state vector is known to lie.

Two senses of "time" have appeared in this discussion. The first is that determined by the background space–time, which enables us to say when one event precedes another, and by how much. But the second sense of time is that of "objective happening," that is, the actual things which mark an event as distinguished. This "objective happening," is, in quantum theory, expressed in the act of measurement. We should take this seriously, and aim to understand it. It provides us with a whole new perspective on the question of what time is.

From this view, time, in the sense of objective happening, is *produced* by, indeed *is* reduction of the state vector. This notion of time is as important as the relativistic one, and the two notions of time should ultimately be united in a single theory.

4 Quantum Field Theory and Measurement

I have so far discussed quantum theory in a general way, without explicitly considering relativistic effects. To reconcile quantum theory with special relativity one passes to quantum field theory. There are several crucial new points which occur.

First, an essential tenet of the theory is that all observable operators be constructed from local quantum fields. It is also generally true that quantum fields are not operator-valued functions on space–time, but operator-valued distributions. This means that an operator must generally be averaged over a space–time volume (against a smooth weighting function); attempts to localize the averaging process lead to more and more wildly fluctuating results, which are manifestations of the divergent zero-point energy — a point I will return to.

Suppose we have such an operator Q, which is the result of integrating the quantum fields (perhaps sums of products of the quantum fields) over a compact space–time volume τ. Using the field equations, we may re-express this same operator in many different ways, as integrals of the quantum fields against suitably evolved weight functions supported on entirely different space–time volumes (for instance, say τ'). Thus the operator Q, considered simply as an operator, is not in any preferred way associated with a single space–time volume. In what sense, then, is the measurement of Q local to τ? When can it be known?

It is natural to assume that any actual measurement must take place via some, as-yet incompletely understood, dynamics which is local in space–time. Thus we think that some sort of a device may be constructed in a neighborhood of the volume τ effecting the measurement of Q, or alternatively, a different sort of device might be constructed on τ' effecting the same measurement. (I am using the term "device" for its intuitive appeal. But it is not meant to suggest that the devices need be manufactured or placed or activated by conscious beings; it may well be that certain physical configurations count as devices irrespective of how they are attained. I shall sketch a suggestion for this below.)

Let us now focus on one such measurement, by the device at τ. When can the result of the measurement be known? The dicta of quantum theory give us no answer to this. On the other hand, causality would seem to imply that the result of the measurement can only be known to observers at events p whose pasts include τ. Thus the process of measurement, comprising reduction together with learning the result, must take a finite time, determined by causality.

While these considerations are very reasonable, there is something of considerable importance for the shape of our conjectured overall theory of time which has crept in: this is that the reduction only comes to be objectively known at *points* p. That is, even though the observable measured might be considerably non-local (a field average over τ, for instance), and even though the reduction may necessarily take place over a significant space–time volume, the actual objective verification of it appears to require a considerable degree of localization in space–time.

It is important to appreciate how strongly this statement depends on some sort of a classical view, to good approximation, of space–time, and how at odds it is with any quantum view of the fabric of space–time itself. If the fabric of space–time had any conventional quantum character, one would think that, besides its usual "position" basis (knowing where events are) there would be, for example a "wave-number" basis as well (knowing the Fourier components describing extended distributions); but it seems hard think of ascribing any definite knowledge of the reduction of a state to a particular wave-number. This is an indication that space–time enters the reduction process in an essentially different way from quantum fields — and this will have implications for reconciling quantum theory with general relativity, an issue which I will return to below.

5 Renormalization, Stress–Energy and Time

A natural approach to understanding time in quantum theory is via its classically conjugate variable, energy. While a great deal of interesting work has been done, for instance, on the time–energy uncertainty relation via quantum-mechanical thought experiments, I want to explore something essentially different here, relativistic effects for which one needs quantum field theory and renormalization. What I shall argue here is that there is good reason to think that time plays a more profound role than has been considered in renormalization and in the meshing of quantum physics to classical space–time. I shall consider here linear quantum field theories.

5.1 Definition of the Stress–Energy Operator

The starting-point for relativistic treatments of the matter-content of a system is the stress–energy, and in quantum field theory this becomes the stress–energy operator. In linear quantum theories, it has formally the same appearance as in the corresponding classical theories, but it is divergent (even as an operator-valued distribution) and must be renormalized. That renormalization is accomplished by *normal-ordering*, that is, re-writing the factors so that no annihilation operator precedes its corresponding creation operator. At a formal level, the normal-ordering subtracts a c-number divergent term, the vacuum stress–energy, from the formal one defined by the local fields:

$$T_{ab}^{\text{renormalized}} = T_{ab}^{\text{local fields}} - T_{ab}^{\text{vacuum}} \,. \tag{1}$$

The normal-ordering on which this is based is very much a temporal concept, for the creation and annihilation operators are defined as the negative- and positive-frequency parts of the fields.

What happens in curved space–time? The construction of the quantum field theory is quite similar; indeed, if one can determine which modes of the field count as positive- and which as negative-frequency, one can mimic the entire special-relativistic construction. This is usually expressed in terms of choosing a complex structure J (with $J^2 = -1$) on a suitable space Γ of classical solutions of the field equations, the positive- and negative-frequency modes corresponding to the two eigenspaces of J.

The choice of J is not unique, however, and so one really has a family of possible quantizations, indexed by the acceptable J's. One requires that the J's lead to n-point functions whose dominant ultra-violet asymptotics are the same as for fields in Minkowski space; this considerably restricts J, to *Hadamard* representations. However, some ambiguity remains.

This ambiguity can be fixed in certain circumstances, such as in a stationary space–time with suitable asymptotics. However, the J's associated with two different stationary regimes will in general be different, being related by a *Bogoliubov transformation*. This means that there is no absolute sense to a creation or an annihilation operator, and no absolute sense to the quantum vacuum. Indeed, it is just this last fact which gives rise to the possibility of an initially vacuum state becoming, without any change, an occupied state — the sense of what "vacuum" and "occupied" are has altered.

The definition of $T_{ab}^{\text{renormalized}}$ thus depends on the definition of normal-ordering, which in turn depends on J, which is ambiguous because of the lack of a preferred time coordinate. Thus in general it is possible for the definition of $T_{ab}^{\text{renormalized}}$ to be ambiguous, although one might hope that in (for instance) stationary regimes a preferred definition of $T_{ab}^{\text{renormalized}}$ would exist. It is known that, if one requires $T_{ab}^{\text{renormalized}}$ to be conserved ($\nabla^a T_{ab} = 0$), then choices of $T_{ab}^{\text{renormalized}}$ do exist which are unique up to a possible conserved c-number addition.

I think it is generally considered that this construction of the stress–energy is essentially correct, and that probably if we are clever enough we will be able to fix the c-number ambiguity. However, there are several points at which I think this view may be mistaken.

5.2 Negative Energies and Measurement

It should first be appreciated that the stress–energy operator, even as an operator-valued distribution, is a rather singular object no matter how the renormalization is done. One would think, by analogy with classical theories, that on a

Cauchy surface Σ the Hamiltonian for evolution along the future-directed time-like vector field ξ^a would be

$$H(\Sigma, \xi) = \int T_{ab}^{\text{renormalized}}\, \xi^a \, \mathrm{d}\Sigma^b \,. \tag{2}$$

This turns out to be true in a weak sense, but $H(\Sigma, \xi^a)$ is in generic circumstances rather pathological (Helfer 1996). It is (unless ξ^a is a Killing vector) not self-adjoint (so it does not have a well-defined spectral resolution and cannot directly be an observable) and it is generically unbounded below (no matter *which* allowable c-number contribution to the renormalization is chosen). If the integral above is replaced by an average over a space–time volume close to Σ, then one can avoid these problems, but as the volume over which the average is taken approaches Σ, the lower bounds tend to $-\infty$. (This occurs even in special relativity, if ξ^a is perturbed by any finite amount from a Killing field.) Thus $T_{ab}^{\text{renormalized}}$ cannot be a conventional measure of energy-content, for it would predict an unstable theory.

We are led to conclude that energy cannot be well-localized in time in quantum field theory. While this behavior is quite different from the non-relativistic time–energy uncertainty relations, it is bound up with them. I have argued elsewhere (Helfer 1998) that it is in effect *symptomatic* of the problem of giving an objective meaning to space–time geometry in the absence of quantum measurement considerations, and that any attempt to verify negative energies' existence directly would likely require a measuring device which would give a net positive local energy — underscoring the lack of objective reality which can be ascribed to such situations, in the absence of quantum measurements.

5.3 Time and Nonlocality of Renormalization

The ambiguity surrounding the definition of the stress–energy has obscured an important issue. It is highly likely that the renormalization prescription forces the stress–energy to be determined by considerations which are *nonlocal in time*. Indeed, in simple cases where one can renormalize in a preferred way, one sees this directly. The complex structure J distinguishes positive from negative frequencies, and this requires a temporal averaging to define. (And the various renormalization procedures used in relativistic quantum field theory are also nonlocal, although the divergent terms are often local.) I should make it clear that this possibility is not usually considered, and indeed, as a way of trying to fix the ambiguity many physicists actually make the opposite assumption (that is, that the renormalization should be local). However, the physical underpinnings of renormalization have very much to do with averaging and considerations of different scales, and so my view is that the more likely possibility is that these nonlocal averages are real.

For linear quantum field theories, these questions about the correct renormalization prescription only contribute finite c-number modifications to the stress–energy. Thus one might think that even if the nonlocal averaging prescription is correct, it only contributes minor technical modifications. However, the nonlocality involved changes the theory in a fundamental way.

The nonlocality would mean that the stress–energy, and hence the Hamiltonian operators generating evolution, cannot be locally known. Thus the energy–momentum-content of a region can at best be inferred only after the fact in light of subsequent developments. I would suggest that this is a new sort of time–energy uncertainty relation. It has in fact a dual character, according to the two concepts of time I have considered: from the usual relativistic point of view, it restricts how well the energy can be known locally; but from the point of view of time as reduction, it means that a reduction measuring such a Hamiltonian must occur over a larger volume of space–time than that indicated by the local operators which go into the stress–energy. (That is, longer than that deduced from general principles in section 4.)

There may be the possibility of observing the consequences of these effects interferometrically. The c-number character of the vacuum term is not absolute; we may imagine quantum operators coupling to it: for instance, if the field interacts with boundaries via Casimir effects, if the precise location of the boundaries is switched according to the quantum state of other operators.

5.4 Quantum General Relativity?

I have already indicated that while it is crucial to integrate the relativistic notion of time with the quantum concept of reduction, there are indications that doing so will treat space–time and quantum theory in different ways. I pointed out that objective verification that reduction has occurred appears to be an inherently *local* concept, which is simply difficult to reconcile with the usual view of quantum theory as being largely independent of notions of localization in space–time.

I also pointed out that measures of energy given by the stress–energy operator are unbounded below, and this suggests that it will be very hard to quantize Einstein's equation

$$G_{ab} = -8\pi G T_{ab} \tag{3}$$

by somehow promoting both sides to renormalized quantum operators and have a physically sensible theory. The unboundedness-below of $T_{ab}^{\text{renormalized}}$ suggests a gross unphysical instability.

And I pointed out that the renormalization of T_{ab} involved the subtraction of a divergent vacuum term T_{ab}^{vacuum}. In the context of linear quantum field theory in curved space–time, this term was a c-number, but if the metric is somehow promoted to a quantum operator, the vacuum term, which depends very much on the metric, becomes a fluctuating q-number which, as we have seen, is moreover likely to be nonlocal in time. This nonlocality moves us beyond the realm of conventional quantum theory.

Thus it seems to me that, while it is absolutely necessary to modify general relativity in some sense to meld it with quantum theory and reduction, there are a number of strong points arguing that the correct modification will not be to simply quantize the metric as another field. Something will need to be done which to some degree preserves the locality of relativity, and it is unlikely that the quantum stress–energy tensors can be in any direct sense considered as sources for whatever the correct extension of general relativity to quantum theory is.

6 What Determines Reduction?

The question of what determines the reduction of the state vector is, as I indicated earlier, a key element which is simply missing from our physical understanding at present. Work on this question is still in very preliminary stages. Here I shall indicate not a particular solution, but what appear to me to be some fruitful avenues for its exploration, together with some consequences.

A prescription for reduction must determine both when it is likely to occur, and what operator is likely to be measured. The fact that the world does not spontaneously reduce to eigenstates of bizarre operators is a considerable restriction on the choice. Also the fact that there is so much freedom in placing the "classical/quantum cut" suggests that by and large the operators which are measured very nearly commute (on the subspaces of Hilbert space in which the states are known to lie) with the Hamiltonians.

These considerations, together with a desire for economy, suggest that the mechanism of reduction should be somehow coded in the Hamiltonians themselves, or in related structure, and should also depend on the subspace of Hilbert space in which the state vector is already known to lie.

While I believe that reduction is likely to occur via non-gravitational forces as well as gravitational ones (presumably, in most physics laboratories most reduction is due somehow to electromagnetic interactions), taking up this issue would lead to consideration of nonlinear field theories. (Preliminary work does indicate that there is indeed scope for applying the ideas here to such theories.) So I will confine my remarks to linear quantum fields in curved space–time. (Similar ideas hold for linear fields in the presence of external potentials.)

We have seen that the Hamiltonians of linear quantum fields

$$H(\Sigma,\xi) = \int_\Sigma T_{ab}^{\mathrm{renormalized}}\, \xi^a \mathrm{d}\Sigma^b \tag{4}$$

are really rather singular objects, being generically unbounded below and not self-adjoint, and that what we really should consider are temporally-averaged Hamiltonians, which would have the general form

$$H(\tau,\xi^{ab}) = \int_\tau T_{ab}^{\mathrm{renormalized}}\, \xi^{ab} \mathrm{d}\tau\,, \tag{5}$$

where τ is a space–time volume in the neighborhood of a Cauchy surface Σ, and now ξ^{ab} encodes both the vector field determining evolution and the normals to the hypersurfaces being averaged over. These averaged Hamiltonians are self-adjoint, and for suitable ξ^{ab} they are bounded below, although as the averaging narrows to a particular Σ the individual lower bounds diverge to $-\infty$.

It seems plausible that what should determine reduction would be a tendency to seek situations in which these Hamiltonians were relatively non-singular. Thus it is possible that (say) reduction becomes likely for those space–time volumes τ and those ξ^{ab} for which the lower bound approaches zero. In this view, reduction over very short times (and the attendant high quantum fluctuations) would be highly suppressed, and also reduction along very "twisted" ξ^{ab} would be highly suppressed; in fact, the reduction would tend to select, as nearly as possible, tensors $\xi^{ab} = \xi^a t^b$, where t^a would be a normal to the surfaces Σ and ξ^a would approximate a timelike Killing vector. It would also be expected that the neighborhood of a Cauchy surface could well develop different domains in which reduction proceeded independently, according as in each domain the criterion for reduction became increasingly likely to be satisfied.

If something like this can be achieved, it would *of itself* provide a definition of a "cosmic time vector" to good approximation, that is, the temporal Killing vector ξ^a which is approximately a component of ξ^{ab}.

7 Conclusion

I have suggested that reduction of the state vector should be considered a sort of time, as important as the conventional relativistic one, and that it is essential to reconcile these notions. From the point of quantum theory, a key question — whether one accepts the rest of the arguments in this paper or not — is, Under what circumstances does a measurement take place?

I have argued that it is likely that the information signalling reduction is somehow largely bound up in the stress–energy operator, and that a correct resolution of that operator's apparently pathological properties might be that they point the way to reduction. A rough indication was given of a general mechanism which might determine both reduction and (to reasonable approximation) a cosmic flow of time.

The problem of reconciling the quantum and the relativistic notions of time gives a point of view on the problem of reconciling quantum theory and general relativity — what is usually called quantum gravity. While this issue is very important and will, in my view, involve some modification of general relativity, it is not at all clear that that modification will be a quantization of the metric in a conventional sense; there are a number of indications to the contrary.

The ideas here are of course only tentative templates for more detailed explorations.

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