

# Might black holes reveal their inner secrets?

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*Black holes harbor a spacetime singularity of infinite curvature, where classical spacetime physics breaks down, and current theory cannot predict what will happen. However, the singularity is invisible from the outside because strong gravity traps all signals, even light, behind an event horizon. In this essay we discuss whether it might be possible to destroy the horizon, if a body is tossed into the black hole so as to make it spin faster and/or have more charge than a certain limit. It turns out that one could expose a “naked” singularity if effects of the body’s own gravity can be neglected. We suspect however that such neglect is unjustified.*

## ***Black holes and cosmic censorship***

Black holes are among the most fascinating concepts in physics. The notion of a gravitating body from which light cannot escape can be traced all the way back to geologist John C. Michell in 1783, but the modern concept to which the term black hole refers arises only within Einstein’s 1915 general relativity theory of gravity. The understanding of these objects came much later, and the term “black hole” came into use only in the 1960s, popularized by John Wheeler.

What exactly is a black hole? Michell’s Newtonian conception of an object whose escape velocity exceeds the speed of light provides a rough understanding of the black hole size, but is qualitatively deeply inaccurate. It describes a situation in which light would rise to a height depending on its initial launch location, and then fall back toward the object. The black hole of general relativity is quite different since, as in special relativity, the speed of light is independent of the conditions of the source of the light. The critical concept needed to describe a black hole is that of the event horizon: a one-way boundary into which objects and light can fall, but out of which nothing, including light or any signal, can come — a boundary in spacetime beyond which events cannot affect an outside observer. Light does not rise up to the horizon and fall back, rather, light at the horizon simply hovers at the horizon. It is the presence of the horizon that motivates the name “black hole”. The horizon itself is just a “surface of no return” in the spacetime, and locally nothing special is happening there. What lies inside the horizon is a wholly different matter however. The black hole harbors a spacetime singularity of infinite curvature — a location where the tidal accelerations characterizing the gravitational field become infinite.

The presence of divergences at the singularity signals the breakdown of general relativity. Einstein’s theory would be unable to predict the outcome of events in the vicinity of the singularity, and unusual phenomena could take place there. To describe these would presumably require a theory that merges gravity with quantum phenomena, a theory has not yet been fully formulated much less understood.

But are spacetime singularities physically relevant or are they simply mathematical peculiarities of special solutions to Einstein’s theory? As a matter of fact, Penrose and Hawking have managed to show that singularities are not only relevant but actually inevitable in gravitational collapse [1, 2]. Chandrasekhar at first [3] and later others had already shown that for spherical matter configurations, gravitational collapse most likely leads to singularities provided that the initial mass is large enough. The importance of Penrose and Hawking’s results lies in their generality, as they do not rely on specific solutions of Einstein’s equations or special symmetry requirements.

So, the collapse of some dying stars, for instance, will inevitably lead to the formation of a black hole containing a spacetime singularity. In this case the presence of the event horizon conveniently hides the singularity from our curious eyes and relieves us from the burden of having to worry about it if all we wish to understand is observations from outside. It would actually be much more exciting if there were no horizon: a visible singularity would give us observational access to the unknown phenomenology mentioned previously. Hence the pertinent question, does an event horizon always hide any singularities? As Roger Penrose first put it in 1969 [4], “does there exist a cosmic censor who forbids the appearance of naked singularities, clothing each one in an absolute event horizon?” The conjecture that such a censor indeed exists is called the “cosmic censorship conjecture”.

The physics of event horizons, unlike that of singularities, should be well described by classical general relativity, so cosmic censorship can be tested and possibly proven without going beyond known physics. Evidence from several directions suggests that any singularity arising to the future of generic (not infinitely finely tuned) non-singular initial conditions, may indeed always be hidden behind a black hole event horizon [5, 6]. However, we are far from having a definitive proof.

Much of the evidence in favor of cosmic censorship comes from the failure of attempts to violate it in thought experiments. Given the difficulty of a direct attack on the general question, this strategy continues to offer an attractive approach to the problem. Even in the context of simplifying approximations, such thought experiments can uncover mechanisms tending to uphold censorship, and they can produce scenarios where censorship would be violated if the approximation were valid. In the latter case, they focus our attention on those “dangerous” scenarios and on the limits of validity of the approximation scheme. This type of approach is what we will be discussing here.

Before going further, it is interesting to mention the implications of naked singularities for black hole thermodynamics, the extension of ordinary thermodynamics to systems containing black holes [7]. According to black hole thermodynamics the area of the horizon of the black hole is proportional to its entropy. In analogy with the second law of ordinary thermodynamics, the area of a black hole can never decrease in a classical process. The role of temperature is played by what is called surface gravity. In analogy with the third law of ordinary thermodynamics the surface gravity cannot be reduced to zero in a finite time. Hawking has provided a proof of the second law of black hole mechanics [8], and Israel a proof of the third law [9]. These proofs assume, among other things, that there are no naked singularities. A violation of cosmic censorship would invalidate both proofs. Moreover, if horizons could be “destroyed” by processes like those discussed in the following, the foundations of black hole thermodynamics would seem to crumble. So there may well be some deep connection between the validity of cosmic censorship and the thermodynamics of spacetime [10].

### *Tilting at windmills*

A much studied obvious strategy for “creating” a naked singularity is to attempt to “destroy” a black hole horizon, that is, to strip the singularity of its clothing. In order to understand this better let us return to the basics of black holes. In general relativity black holes are fully characterized by three quantities: their mass  $M$ , spin angular momentum  $J$ , and electric charge  $Q$ . The spacetime in the vicinity of the black hole is described by the Kerr–Newman (K-N) metric — the measure of distances and times — which contains the three parameters  $M$ ,  $J$  and  $Q$ . The K-N metric describes a black hole as long as the mass is sufficiently large compared to a combination of the charge and angular momentum,  $M^2 \geq a^2 + Q^2$ , where  $a = J/M$ . (We adopt units with Newton’s constant  $G$  and the speed of light  $c$  both set equal to unity. Displaying hidden factors of  $G$  and  $c$ , the quantities  $MG/c^2$ ,  $a/c$  and  $Q\sqrt{G}/c^2$  all have the dimension of length.) The case where  $M^2 = a^2 + Q^2$  is called an extremal black hole, while for  $M^2 < a^2 + Q^2$  there is no event horizon and the K-N metric actually describes a naked singularity. Therefore, it would naively seem that to create a naked singularity all one need do is to start with a black hole and toss in matter with enough angular momentum or charge so as to drive its parameters beyond the extremal limit, leaving it no option but to expose the singularity.

On further thought there are some subtleties involved in such a scenario. First of all, the K-N metric describes stationary configurations, so the proposed strategy can work as stated only if, after having absorbed the matter, the system settles down to a stationary configuration containing all the mass, angular momentum and charge, i.e. without having shed the excess angular momentum or charge in the settling down process. Such an outcome is by no means guaranteed. Indeed, it seems rather unlikely, given the evidence that the trans-extremal K-N metric is unstable [11, 12]. If such instability occurs (the uncertainty lies in the proper boundary conditions at the singularity), the system may well not settle down to such a metric, and at present nobody knows what it would do. What this means is that to demonstrate the creation of a naked singularity one would have to follow the evolution further than the initial “absorption” of the extra matter. So the possibility of initially overspinning or overcharging an initial black hole configuration can only be taken as an indication that cosmic censorship *might* fail.

A second subtlety is that the notion of “exposing” the singularity may be inappropriate, since the singularity inside a perturbed charged or rotating stationary black hole cannot send signals to any point, even those interior to the horizon. (Here we assume that the Cauchy horizon inside the black hole is indeed unstable, as evidence indicates [13].) That is, it has no nonsingular future. Hence it is not so clear that, if a horizon could be “destroyed”, the result would be to expose the singularity that would have been there had the horizon *not* been destroyed. It might produce a wholly different singularity. Nevertheless, either way the process would violate cosmic censorship.

To follow the evolution exactly is a very difficult problem that presumably requires numerical solution of the Einstein equation. The difficulty arises because general relativity is a highly non-linear theory, but even linearizing around a background black hole solution remains a very difficult problem. So most studies of cosmic censorship to date have been carried out in the simple framework of the “test-body approximation” in which the matter being tossed into the black hole moves in a way determined by the original black hole metric (and electric field, if present). This approximation does not take into account effects arising from the gravitation of the matter itself, namely the gravitational radiation and the self-force. To try to justify the test-body approximation one can assume the conditions

$$\delta E \ll M, \quad \delta J \ll M^2, \quad \delta Q \ll M, \quad (1)$$

where  $\delta E$ ,  $\delta J$ , and  $\delta Q$  denote the energy, angular momentum and charge of the body.

Now, provided the body can be tossed into the black hole, the final composite object would have mass  $M + \delta E$ , angular momentum  $J + \delta J$  and charge  $Q + \delta Q$ . In order for the K-N metric with these parameters to be a naked singularity they would have to satisfy the inequality

$$(M + \delta E)^2 < \left( \frac{J + \delta J}{M + \delta E} \right)^2 + (Q + \delta Q)^2. \quad (2)$$

Various special cases have been considered in the literature. For instance, one could consider only charge or only angular momentum. Alternatively, one could start with charge but no angular momentum for the black hole, and then try to reach a trans-extremal state by adding angular momentum but no charge, or vice versa.

In nature, black holes tend to increase their angular momentum by accreting matter, whereas they tend to decrease their charge by attracting opposite and repelling same charges. For a point of principle, one may perhaps ignore these facts, but nevertheless it would be much more provocative and promising if arguments showed that a naked singularity could be created using only angular momentum, since that might then actually occur in nature. Therefore, we choose to focus here mostly on the spinning black hole case. We shall demonstrate that in the test body approximation a trans-extremal condition can be attained, even when taking into account constraints on the size and structure of the body. The limitations of the test-body approximation will then be addressed.

With  $Q = \delta Q = 0$ , the inequality in eq. (2) takes the form

$$J + \delta J > (M + \delta E)^2. \quad (3)$$

This yields a lower bound on the required angular momentum carried by the body, for a given energy  $\delta E$ :

$$\delta J > \delta J_{min} = (M^2 - J) + 2M\delta E + \delta E^2. \quad (4)$$

Since we are assuming  $\delta E \ll M$ , it might seem that the  $\delta E^2$  term may as well just be neglected at this stage. However, as we will see later on, the presence of that term imposes an upper bound on  $\delta E$  and  $\delta J$  and, therefore, should not be neglected.

We can already extract a useful piece of information from this last equation. Dividing both side by  $\delta E^2$  and observing that each term on the right hand side should by itself be smaller than the left hand side, we get

$$\delta J / \delta E^2 > 2M / \delta E \gg 1. \quad (5)$$

If  $\delta E$  were equal to the rest mass of the body (it can be much less if the body is deeply bound by the gravitational field of the black hole), and if  $\delta J$  comes from spin (rather than orbital angular momentum), this would imply that the body has angular momentum far over the extremal ratio. In that case the body could not be a black hole. This does not mean that it would have to be a naked singularity itself, as there is no a priori upper limit to this ratio for bodies other than black holes. Stars for instance can easily have ratios much larger than 1.

We have got a lower bound on the angular momentum of the body from the requirement that the composite object be a naked singularity.

An upper bound comes from the requirement that the body does indeed cross the horizon and end up in the black hole — or rather what was formerly the black hole. To check whether this requirement is satisfied one may resort to

the equations of motion for the body [14]. These are the Papapetrou equation if the body's angular momentum is due to spin, and the geodesic equation if it is orbital angular momentum. But a simpler and more transparent method is to just consider the flux of energy and angular momentum into the black hole when the body falls across the horizon. The requirement that the energy momentum tensor satisfies the null energy condition (which follows for example if the energy density is positive in all local reference frames) yields [15]

$$\delta E \geq \Omega_H \delta J, \quad (6)$$

where  $\Omega_H = a/2Mr_+$  is the angular velocity of the horizon and  $r_+ = M + (M^2 - a^2)^{1/2}$  is the horizon radius in Boyer-Lindquist coordinates. This condition can be written as

$$\delta J \leq \delta J_{max} = \frac{2Mr_+}{a} \delta E. \quad (7)$$

It guarantees that the body can fall across the horizon starting from *some* point outside, although in general the body is in a bound orbit that does not come from spatial infinity.

We now have both an upper and a lower bound for the angular momentum of the body, for a given energy. As long as  $\delta J_{min} < \delta J_{max}$  for some  $\delta E$ , there will be values of  $\delta J$  and  $\delta E$  satisfying both inequalities (4) and (7). First let us suppose the black hole starts exactly extremal, i.e.  $J = M^2$ . Then  $a = M = r_+$ , so one has  $\delta J_{min} = 2M\delta E + \delta E^2$  and  $\delta J_{max} = 2M\delta E$ . This implies that  $\delta J_{min} > \delta J_{max}$  so it is impossible to over-spin the black hole. This was observed long ago by Wald [14] (via a significantly more involved calculation). The physical interpretation is the following: In the case of the spinning body, the spin-spin interaction with the spin of the black hole is sufficiently repulsive to prevent the body from falling in if it would have overspun the black hole. In the orbital angular momentum case, the impact parameter of the body is too large for it to hit the horizon if it has the angular momentum required to overspin.

In the sub-extremal case, however, the inequalities *can* be satisfied, as was shown in [15]. The limiting case where the body is dropped from a point on the horizon had been considered previously by Hod [16]. To understand the range of overspinning parameters, it is helpful to visualize the inequalities graphically. If  $\delta J_{max}$  and  $\delta J_{min}$  are plotted vs.  $\delta E$ , the former is a straight line through the origin, with slope  $2Mr_+/a > 2M$ , while the latter is a parabola with positive intercept, slope  $2M$  at the intercept, and curved upwards. Some algebra reveals that the parabola always intersects the straight line in two points. The allowed values of  $\delta E$  and  $\delta J$  are those in the compact region above the parabola and on or below the straight line. Note that if the  $\delta E^2$  is neglected in (4), the parabola is replaced by a straight line, and no upper bound is imposed on the allowed values. The case considered by Hod [16], i.e. that of dropping the particle from a point on the horizon, corresponds to the upper boundary of this region,  $\delta J = \delta J_{max}$ .

To quantitatively characterize the overspinning region we can expand in the small dimensionless quantity  $\epsilon \ll 1$  defined by

$$J/M^2 = a/M = 1 - 2\epsilon^2. \quad (8)$$

(Hubeny [17] used the same parameter to analyze the charged case, see below.) The parameter  $\epsilon$  measures how close to extremality the black hole is to begin with. It is now useful to adopt units with  $M = 1$ , to keep the expressions simpler. Then we have

$$\delta J_{min} = 2\epsilon^2 + 2\delta E + \delta E^2 \quad (9)$$

$$\delta J_{max} = (2 + 4\epsilon)\delta E, \quad (10)$$

where terms of order  $O(\epsilon^2\delta E)$  have been dropped in (10). The allowed range of  $\delta E$  lies where the difference

$$\delta J_{max} - \delta J_{min} = -2\epsilon^2 + 4\epsilon\delta E - \delta E^2 \quad (11)$$

is positive, i.e.

$$(2 - \sqrt{2})\epsilon < \delta E < (2 + \sqrt{2})\epsilon. \quad (12)$$

In particular,  $\delta E$  must be of order  $\epsilon$ , which is consistent with the requirement (1) that the body make only a small perturbation. For a given  $\delta E$ , the allowed values of  $\delta J$  are near  $2\delta E$ , so we must have

$$\delta J \sim \delta E. \quad (13)$$

Note that the *width* (11) of the allowed range of  $\delta J$  is only of order  $\epsilon^2 \ll \epsilon$ .

Clearly, the black hole must start out very nearly extremal. To be somewhat more quantitative, according to (1) and (12) we must have  $\epsilon \ll 1$ , and  $a - 1 = 2\epsilon^2$  is parametrically smaller. For example, if  $\epsilon = 10^{-2}$ , then the initial black hole must have  $a = 0.9998$ . For a thought experiment, we can imagine even smaller values of  $\epsilon$ . We conclude that, if the body can be treated as a point test particle moving on the background spacetime of the black hole, the black hole can indeed be over-spun!

A similar conclusion had been previously reached by Hubeny [17], for the case of adding charge to a charged black hole ( $J = \delta J = 0$ ). In analogy to the spinning case, in the charged case the two constraints are

$$\delta Q > M - Q + \delta E, \quad (14)$$

$$\delta Q \leq \frac{r_+}{Q} \delta E, \quad (15)$$

where now  $r_+ = M + \sqrt{M^2 - Q^2}$ . If the black hole starts out extremal,  $M = Q = r_+$ , then overcharging is impossible. However if  $M > Q$ , then  $r_+ > Q$ , and one easily sees by visualizing the inequalities graphically (now they are both straight lines) that there is an infinite range of solutions, once  $\delta E$  is greater than a certain minimum value.

A similar conclusion was also reached by de Felice and Yu [18], for the case of adding angular momentum to an extremally charged black hole ( $Q = M$ ,  $J = \delta Q = 0$ ). In this case the minimum  $\delta J$  to overspin is given by

$$\delta J > (M + \delta E) \sqrt{2M\delta E + \delta E^2}, \quad (16)$$

and there is no maximum  $\delta J$ , since the only requirement for the body to fall across the horizon is  $\delta E \geq 0$ , which does not involve  $\delta J$ . Note that to lowest order in  $\delta E/m$  the minimum overspinning  $\delta J$  is  $\delta J_{\min} = M\sqrt{2M\delta E}$ .

### *Does size matter?*

So far we have characterized the body only by its energy  $\delta E$ , angular momentum  $\delta J$  and charge  $\delta Q$ . We did not consider restrictions placed on its size and structure: It should be sufficiently small to justify use of a test particle approximation, and it should be composed of matter having positive energy density and no unphysically large stresses. The next step is to take into account these issues, which we shall do here for the case with no charge [15].

We begin with the spinning case. For simplicity we assume that the body is dropped along the rotational axis of the black hole. We first consider the case where the body has  $\delta E \sim m$ , and is not spinning relativistically, so its spin angular momentum is given by  $\delta J \sim mvR \sim \delta E vR$ , where  $v$  is the surface velocity and  $R$  is the equatorial radius. The condition  $v < 1$  then implies  $R > \delta J/\delta E$ . We saw above that the ratio  $\delta J/\delta E$  must be of order unity (13), that is of order  $M$ . In this case the body must be larger than the black hole, so it simply will not “fit” in the transverse direction, and in any case treating it as a point particle with spin would be unjustified, since that rests on the smallness of the size of the body compared to the ambient radius of curvature. Moreover, one can show that the radial tidal stress required to hold the body together would be larger than the energy density, violating energy conditions. It cannot help to allow ultra-relativistic tangential velocity: as a simple Newtonian estimate shows [15], that would require unphysical stresses holding the body together. The conclusion is that it is impossible to over-spin the black hole if the body’s energy is close to its rest mass,  $\delta E \sim m$ .

Since the angular momentum involves the rest mass  $m$ , not the energy  $\delta E$ , it might be possible to achieve a large enough  $\delta J$  with a small enough size  $R$ , without requiring unphysical matter, by dropping the body from a position where it is deeply bound,  $\delta E \ll m$ . This might be achieved by slowly lowering the body on a tether, down to the near the black hole horizon, before dropping it in. Now we reconsider whether the size restrictions can be met in this setting.

We begin with the restrictions on the rest mass  $m$ . If  $m$  is much greater than  $\delta E$ , then the test body approximation requires that we impose not only  $\delta E \ll 1 (= M)$  (1), but also  $m \ll 1$ . There is also a lower bound on  $m$ , coming from an upper bound on  $R$ : the angular momentum is  $\delta J \sim mvR$ , hence (restricting to nonrelativistic spin  $v < 1$  as required by the previously mentioned result)  $R > \delta J/m \simeq 4\epsilon/m$ . The requirement  $R \ll 1$  then yields  $m \gg \epsilon$ . The mass and size must therefore fall within the ranges

$$\epsilon \ll m \ll 1, \quad 4\epsilon/m \lesssim R \ll 1. \quad (17)$$

To these conditions we must add the requirement  $R \gtrsim m$  that the body is not a black hole, as explained above.

The inequality (6) guarantees that the body can cross the horizon with the chosen values of energy and angular momentum, but since the deeply bound drop point lies at a finite distance from the horizon it is necessary to check that (a) the spinning body would actually fall into the black hole rather than being repelled, and (b) it is possible to choose the polar radius of the body  $R_{\text{polar}}$  to be smaller than the proper distance  $d$  from the horizon to the drop point

$$R_{\text{polar}} < d, \quad (18)$$

so that it can fully “fit” outside the black hole and be localized at the drop point.

In order to fall in, the maximum value that  $d$  can have, given the allowed values of  $\delta E$  and  $\delta J$ , turns out [15] to be

$$d_{\text{max}} \simeq \epsilon/m. \quad (19)$$

Thus we arrive at the bound

$$R_{\text{polar}} < \epsilon/m. \quad (20)$$

Together with (17), this means that the body must be at least somewhat oblate,  $R_{\text{polar}} \lesssim R/4$ , but not exceedingly so [19]. We conclude that the body can be large enough to possess the requisite angular momentum with a physically acceptable stress, and still fit outside the black hole at the drop point [20].

We now turn our attention to the case of orbital angular momentum in the equatorial plane. Here the issue is that in order to have the required values of  $\delta E$  and  $\delta J$ , the body might have to be in a bound orbit, which would have a turning point at a maximum radius. In that case we would need to require that the body be small enough to fit outside the horizon at this radius. Since the body can be no smaller than a black hole with the same rest mass, it is not clear in advance whether this requirement could be met. However, as has been shown in [15] this size constraint is not an issue, since in fact there are suitable orbits that come in from infinity with no turning point. This can be shown numerically, but also analytically by the use of the effective potential governing the motion of a test particle in a Kerr spacetime (Kerr-Newman with no charge) [15].

Let us briefly consider the size and structure requirements when attempting to overcharge or overspin a charged black hole. In the case with no angular momentum Hubeny [17] showed that the body can have the required charge and mass, with low internal stresses and size much smaller than the black hole. Also, she demonstrated that there are charged test particle trajectories that fall from infinity into the black hole. Therefore, much like the orbital angular momentum case, size constraints are not an issue. On the contrary, for spinning particles dropped radially with radial spin into an extremal charged black hole, de Felice and Yu [18] found that the test body must be bound very close to the horizon. The same is true for a particle carrying orbital angular momentum but no spin, as we now show. The radial motion is governed by the equation  $\dot{r}^2 + (1 - 1/r)^2(1 + \tilde{L}^2/r^2) = \tilde{E}^2$ . Here  $\dot{r}$  is the derivative of the Reissner-Nordstrom radial coordinate with respect to the particle proper time, and  $\tilde{E} = (\delta E)/m$  and  $\tilde{L} = (\delta J)/m$  are the energy and angular momentum divided by the particle rest mass  $m$ , and we have again set  $M = 1$ . As mentioned after (16), the overspinning requirement is  $\delta J/\delta E \gtrsim \sqrt{2/\delta E} \gg 1$ , hence  $\tilde{L}/\tilde{E} \gg 1$ . For an unbound orbit  $\tilde{E} \geq 1$ , so we infer that  $\tilde{L} \gg 1$ . There are therefore two turning points where  $\dot{r} = 0$ . To fall into the hole the particle must lie inside the inner turning point, which lies at a radius  $r_{\text{inner}}$  very close to the horizon, where  $r_{\text{inner}} - 1 \simeq \tilde{E}/\tilde{L} \lesssim \sqrt{\delta E/2} \ll 1$ . However, although the radial coordinate must be very close to that of the horizon, the proper distance to the horizon, measured in the static frame, is infinite for an exactly extremal black hole. Hence, in both the spin and orbital cases, there is no need to further consider size constraints that might have been imposed by the location of the turning point (see also footnote [20]).

### *Including the body's own gravity*

But what about gravitational wave radiation and self-force, which our approximation neglects, as mentioned previously? In fact, a body orbiting a black will always lose energy and angular momentum in gravitational radiation. Additionally, it will always experience self-force effects due to the “refraction” of its own field on the background of the black hole. We saw that, from purely kinematic considerations, the relation between the energy and angular momentum of the dropped body must be very finely tuned: they are both of order  $\epsilon$  in magnitude, but the allowed

window for angular momentum, given the energy, is only of order  $\epsilon^2$ . Since the over-spinning process we found involves a delicate balance, it is certainly possible that, although small, gravitational radiation and/or self-force effects may always manage to preclude the over-spinning.

Given that the inequalities (4) and (7) need only hold on the horizon, one could imagine that the loss of energy and angular momentum in gravitational radiation could be compensated by adjusting the initial conditions. In the case of an axially symmetric spinning body there is no radiation of angular momentum, so it should be possible to simply compensate for the energy radiated. To determine whether compensation is actually possible in the orbital case requires further investigation. More worrisome is whether it is possible to overcome the self force effects. Note that Hubeny found strong indications that for the charged case the electromagnetic self-force might indeed prevent the overcharging, although her calculations were not conclusive [17]. The gravitational radiation and self-force effects can in principle be calculated numerically, at least in the linearized theory, but the calculation would likely be extremely challenging.

Another distinct effect that might prevent the creation of a naked singularity is the tides raised on the black hole horizon by the falling body. These would be irrelevant for the orbital angular momentum case since the body falls in from spatial infinity. In the spinning body case, however, in which the body is lowered to the horizon and then dropped, the tidal bulge of the horizon might perhaps make it impossible for the body to start out in the exterior while still satisfying the size constraints.

Given the existing evidence for cosmic censorship, it seems indeed likely that neglected gravitational effects will come to its rescue. Our analysis suggests a dynamical regime in which it may be interesting to study these neglected effects. It turns out that what is ultimately possible in physics hangs very much on the details when it comes to black holes...at least until the day that we acquire a deeper understanding of the uncanny tendency of general relativity toward cosmic censorship.

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- [19] In [15] the possibility that  $R_{\text{polar}} \neq R_{\text{equator}}$  was overlooked, so it was erroneously concluded that no value of  $R$  could meet all requirements.
- [20] de Felice and Yu made a similar analysis in the case of dropping the spinning body into an extremal charged black hole, but they computed the coordinate radius corresponding to  $d_{\text{max}}$ , rather than the proper distance. In the extremal case, the proper distance to the horizon is infinite in the direction orthogonal to the Killing vector, so there is apparently no requirement that the body be disk-shaped, contrary to what was stated in [18].