Let's Call The Whole Thing Off/On

Alexander Lamb
alex.lamb@gmail.com
builtuniverse.wordpress.com

Abstract

We propose that science should proceed under the assumption that Nature is discrete unless discrete models are proved untenable. We outline an argument that explains the reasoning for this position. We also describe examples of simple quantized mechanisms which give reason to believe that the discrete approach can capture all the observed symmetries of nature.

1 Introduction

Floridi and others have pointed out that we can never know with philosophical certainty whether the universe is continuous or discrete [1]. For any continuous model we can construct, we cannot rule out the possibility that some discrete model underpins it at a scale that experiment will never let us probe. Similarly, for any discrete model of the universe, we cannot rule out the possibility that the underlying system is fundamentally continuous.

However, science is not a search for philosophical certainty as it lies outside the domain of testable predictions. We propose instead that science is a search for tools, and that, broadly speaking, there are two kinds of tools that science seeks out: tools of prediction and tools of understanding. Neither is adequate in isolation. A tool that predicts events without delivering insight as to why they occur is a black box that impedes the development of better tools. A tool that enables one to 'understand' Nature, but predict nothing is useless as its assertions can never be tested.

The direction that science proceeds should therefore be the one that maximizes the usefulness of the tools it develops. The most effective tools are those that provide the most insight, and allow the most accurate prediction, with the least amount of computational effort. Such tools can be said to be efficient, or 'elegant'.

Though philosophical certainty may elude us, it is still safe to say that if we believe in the validity of the scientific endeavor, we also believe that there will be some set of rules that can model everything we witness in the universe and that no further level of description is required. In this respect, we expect that Nature must at some level correspond to the requirements of the Mathematical Universe Hypothesis [2], and that further speculation is irrelevant.

However, examining Nature at this fundamental scale necessitates a difference in approach. When attempting to match the totality of phenomena that Nature can generate, the right tools for the job can only be as efficient as the system they seek to describe. No amount of compression in the representation can exist, otherwise details will be missed. The beauty, or lack thereof, in the theory ceases to be relevant. It is also safe to say that this set of rules will require either discrete or continuum mathematics.

In this essay, we attempt two goals. The first is to make it clear why we should expect the universal rule-set to be discrete whether we prefer this model or not. The second is to outline attempts to falsify the discrete approach, and to demonstrate that, so far, discrete models appear capable of capturing all the key symmetries required of a full physical theory.

2 The case for discrete Nature

Many in the computational physics community have characterized the reason for preferring a discrete model for the universe as due to Ockham's Razor. However the use of this expression has lead to some disagreement and confusion. Ockham's Razor is a tool for selecting between two theories that fit some body of experimental evidence. It encourages a scientist to select the simpler of two models so long as both do an equivalently good job of matching the data. It is considered a heuristic tool, not a law, and not one that always prefers the simplest answer, as in many experimental cases the simplest model fails to fit all the data.

In order to make the case for a discrete universe clear, a new term is required along with a clear explanation of its meaning. We use the term: *Principle of Minimal Complexity.* This principle states that given a set of complete and correct physical theories that explain every event in the universe to date, we should prefer the theory that has the smallest Kolmogorov complexity. In other words, the theory for which the rules plus the data required to describe it are the least. The reasoning is as follows.

Let us consider a list of all complete and correct models of Nature along with the data they require, ordered by complexity. There will be more than one such model, as some will contain redundancy in their description. Furthermore, the models in the list may predict very different sets of future events despite agreeing on the past, as some will contain extra information that only becomes relevant at a future point. It is simple to see that the less information a model requires, the more likely it is to reappear later in the sequence under some more redundant guise. Thus, models that predict that the universe will continue much in the way it has so far will occur with higher frequency than those requiring that grand pianos materialize on every street-corner next Tuesday. In fact, we should expect that the simplest such model will occur very much more often than all others. Given that our reality must match a model somewhere in this list, we propose that we inhabit a reality that corresponds to the simplest possible such model with probability tending to one. This fits well with what we see of physical law. The universe generally behaves in a predictable way, even if the outcomes of individual microscopic events are uncertain. Determinism, and therefore minimality, appear to be features of Nature.

However, it is also the case that if a discrete model occurs in the list, it will precede the continuum models. This is because any particle property that varies smoothly cannot be represented in a countable number of discrete bits, whereas a single continuous number can code for any number of discrete values at the same time. Furthermore, any complete model must encode all the information in the functions it employs, not just its data. While there are smooth measures for information that operate on data, the author is aware of no such measure that operates on models without yielding infinities. Thus, regardless of the measure of complexity we employ, continuous models contain more data. We can therefore expect discrete models to occur in the sequence more often than continuous ones. Indeed, if the series contains *any* workable discrete model, it will be infinitely more likely to be true than a continuous one.

Though the idea of an infinite list of correct theories may, on the face of it, not appear relevant to the practical task of physics research, the alternative is to believe that the simplicity of a model is not important. In this case, a theory that predicts grand pianos for all has equal validity to any that do not incorporate this feature. This is surely not useful from the standpoint of experimental prediction.

One might also propose that this argument self-selects for a discrete system by requiring that the terms in the list under examination be dynamical models of Nature. Arguably, some other descriptive set, such as the set of all representations of the current moment in time, could be examined and regarded as equally valid for consideration. Any such set lacking repeated terms would select for a more complex description of reality rather than a minimal one. However, this rebuttal requires that we accept that the universe is arbitrarily complex and that no useful statements about the outcomes of experiments can be made. This is because choosing a random element from such a set necessarily also entails selecting an arbitrary amount of extra information about the universe's future. In other words: grand pianos again. In essence, expecting the universe to behave predictably is tantamount to preferring a discrete representation if one exists. There is no clear way to prefer simple continuous representations over both simpler discrete ones and more arbitrary continuous ones, so long as all theories are correct.

It is for this reason that many computational theorists tend to invoke Ockham's Razor. The preference for simplicity which guides the scientific process, and which leads us toward elegant solutions, *necessarily* leads us to prefer those systems that make no use of continuous numbers, if such examples exist [3].

This reasoning is of course only relevant if a discrete model of physical systems can be found. A proof that such a model cannot exist would be tantamount to ruling out the discrete approach. Section 3 outlines the reasons to believe that such a proof will not be possible.

Before we proceed, however, it is worth acknowledging that while working discrete theories may be found, it may be that no such theory turns out to be useful for making reliable experimental predictions. Continuum theories work well at the scales we can probe, and it may be that discrete theories can only work at scales that make the computational burden of prediction intolerable. In this case it would be tempting to rule out discrete models as irrelevant from a practical standpoint. However, in this case, knowing that the predictive theories *must* converge onto a discrete model at some scale may aid in the production of better predictive theories, as those dependent on a continuum may be ruled out. Fortunately, as the examples in the following section illustrate, we have good reason to hope that discrete models will provide us with plenty of experimental predictions.

3 Discrete models of smooth symmetries

All experimental evidence of the physical world to date suggests that some number of apparently continuous symmetries are required for its description. Not only are these symmetries smooth to all measurable scales, but in combination they appear, at first glance, to be incompatible with the idea of a discrete model.

Given this rather unpromising starting point, we propose that in order to determine whether a discrete model for Nature is possible, we examine each such symmetry in turn and determine whether it is, in reality, incompatible with discrete representation. We do this via 'computer-aided thought experiments'. By this we mean that we consider the behavior required of a given symmetry, attempt to discretize it, and consider the consequences in action via the use of software simulation.

We ask: *when discretized, can the experimental properties of the symmetry in question be retained?*

Such thought experiments are not intended to duplicate physical results exactly or act as predictive models. Instead, they are designed to demonstrate that the symmetry under consideration can be captured using the limited set of descriptive tools available to discrete models. If such a thought experiment succeeds, it implies that we cannot rule out the possibility that the phenomenon under consideration is discrete in Nature.

If all the symmetries we would like to model can be obtained in isolation, a second sequence of thought experiments is proposed that would incrementally combine the effects. If a final, integrated thought experiment is eventually derived, this might form the foundation for a theory of Nature. However, a thought experiment of this caliber has yet to be achieved.

In proposing such thought experiments, we inevitably draw on examples from our own research. However, this should not be interpreted as implying that the simulations described below are in some way special or represent approximations to a finalized physical theory. They are used merely because they fit our requirements. We use our own simulations because the number of examples in this research domain is currently very small.

To summarize the results to follow, the following properties of physical systems have been captured in isolation: rotational and translational invariance, temporal invariance, Lorentz invariance, intrinsic angular momentum, particle self-interference, spatial distortion, temporal effects due to spatial distortion, particle variety corresponding to group symmetries and scale-dependent symmetries such as those required to describe the Higgs field. Most of these features exist in an easily combinable format, and some have been successfully merged, though much work remains to be done in embedding all these features in a single simulation.

3.1 Rotational, Translational, and Temporal Invariance

In [4], we outlined a simple model through which the symmetries required for conservation of energy and momentum could be derived in a discrete system. In this model, both space and the experimental 'pseudo-particle' used to generated the desired behaviors are represented using nodes in a densely-connected irregular graph. With each iteration, the neighbor relations for the particle node are rewritten. For the purposes of this experiment, the neighbor relations of the spatial nodes are held constant.

In order to test that rotational and translational invariance are achieved, we purposefully create a graph corresponding to a patch of flat space-time by sprinkling nodes with random coordinates onto a background manifold. We then connect nodes together if their Euclidean distance across the manifold lies below some threshold linking radius r. We use an irregular graph to ensure that the bulk properties of the graph remain identical regardless of the orientation of the pseudo-particle.

To represent the pseudo-particle, we select two sets of nodes of fixed size k at random from the total set. These sets represent the 'front' and 'back' nodes for the particle. The particle is then iterated with each step of the simulation using the iterative relation shown below, with the following formalism: Set S of size m has members $x_1, x_2 ... x_m$. The function $Neighbors(S)$ indicates the set of all nodes that are linked to by any member of S. We also define the function $Top(n, S, f(x))$, which returns the n top scoring members of S, using the metric $f(x)$ to score each element x.

$$\begin{aligned} A' &= Top(n, Neighbors(A \cup B), |Neighbors(x) \cap A| - |Neighbors(x) \cap B|) \\ B' &= A \end{aligned}$$

When this simulation is run, the nodes representing the pseudo-particle become tightly grouped in one area of the graph within a few iterations. The groups align with random orientation and the pseudo-particle begins to move in a straight line, creating the appearance of a physical object. This object remains in permanent motion at stable velocity and is as likely to travel in one direction as any other. While the initial particle localization is not reversible, all subsequent behavior is.

This phenomenon is robust and extensible in that it can be used as the foundation for other speculative simulations in arbitrary numbers of dimensions [5]. Among the other properties that have been captured using this approach are polarization, intrinsic angular momentum, and particle variety adhering to group symmetries.

Such software toys are appealing in their simplicity but cannot be considered analogs of physical particles as they lack properties such as wavelength, mass, Lorentz invariance, and the following of all paths in quantum superposition. In order to achieve such goals, somewhat more subtle thought experiments are required.

3.2 Special Relativity

In [6], a proof is provided that a fully discrete model of Nature employing both a discrete approximation to Minkowski space, and quantum correlation via non-local graph linking, cannot capture both quantum and relativistic effects. This is because quantum detections that are encoded explicitly in a space-time structure impose a strict ordering on events, and for any such ordering there will be some frame of reference in Minkowski space for which we would expect to see the ordering reversed.

This proof demonstrates that in developing a discrete model of Nature we have two choices. The first is to abandon non-local linking and find another mechanism for permitting quantum correlation. This option is under investigation by the Causal Set Program. It appears hard, though perhaps not impossible, to resolve this without resorting to the complex variables of traditional quantum mechanics. The second option is that we avoid modeling Minkowski space directly.

At first glance this second option appears at odds with physical fact, as experimental evidence provide ironclad evidence that the Minkowski metric defines our experience of space-time to the smallest levels of accuracy testable. However, there is an important distinction here: the Minkowski metric defines our *experience* of space-time, not space-time.

The Minkowski metric, in fact, comprises the most succinct in a hierarchy of representations of space-time incorporating ever increasing levels of redundancy. Which of these representations corresponds to reality can never be ascertained by experiment, as, *in order to measure space-time, we must always pass something through it.* Direct examination of the fabric of space-time is impossible as experiences of particles within it are the only guides available. This hierarchy of representations allows us to select one in which long-distance graph associations can be used, and in which all necessary relativistic effects are obtained.

One such representation is given by adding a single compact dimension to space-time that codes for the space-time interval s. The simplest such model of this sort, using a single physical dimension x, requires three dimensions for representation: x, t, and s. Pseudo-particles in such a simulation travel some fixed distance with each moment that has some component in the x direction, and some component in the s direction, along with a fixed distance in the t direction. Distance traveled in the s direction measures the pseudo-particles subjective experience of time, while t represents ‘global time’, a value not measurable by observers within the simulation. The discrete version of this model uses iteration steps to encode the t direction, and embeds s and x in a spatial graph.

An extension of the simulation described in 3.1 to incorporate this extra dimension demonstrates relativistic effects (see appendix for video). While this depiction of special relativity may not look familiar to those used to visualizing the phenomenon in the absence of Euclidean directions, the mathematics are identical. Furthermore, the addition of extra spatial dimensions y and z to the model does not require extra compact dimensions–one still suffices. This model also provides the effect of Lorentz contraction although it is not apparent on external observation until measurements within the simulation are made.

This experiment succeeds because different reference frames within the simulation are represented by different orientations of pseudo-particles with respect to the compact dimension s. From within the simulation, all such frames are identical. It is only from outside that a Euclidean dimension is visible. However, this is all that is necessary in order to incorporate quantum effects via direct linking, as described in section 3.3. We have circumvented the limitation described in [6] by decoupling the subjective experience of time for each particle from the number of iterative steps that the global system has taken. By constructing a new graph in which we link the subjective, time-ordered experiences of a large number of interacting particles, it should be possible to derive a Lorentz-invariant causal set. In doing so, we can demonstrate that the experience of space-time conforms to Minkowski space as expected.

This approach can be improved significantly by employing local symmetry instead of a compact dimension. It is possible to construct a pseudo-particle model such that relativistic effects are derived from a sub-particle following a helical path within a parent particle. In this case, the helical orientation of the sub-particle defines the frame of reference of the parent. Subjective time-steps in the parent of the sort required by special relativity can be easily derived from periodic interaction with the sub-particle. This approach is attractive from a computational perspective, as it supports the exploration of other symmetries using the same mechanism, such as those required to model the Higgs field.

3.3 Quantum Mechanics

In order to create a working approximation of the double-slit experiment, we use an irregular spatial graph of the sort used in 3.1, but which has had a 'screen' cut into it featuring two slits. The only places where the spatial graph on one side of the screen is connected to the other is via nodes and links passing through the slits. The upper half of this graph contains a specific node where we introduce each test particle. The bottom half of the graph contains a strip of nodes at the far end marked as 'detectors'. In order to permit variations to the experiment, such as placing a detector at a slit, we allow for any node in the graph to be marked as a detector. We also mark some small random subset of the nodes scattered across the graph as 'virtual event sites'–a term that is explained below.

To model self-interference, we use a pseudo-particle comprised of a set of 'expanders'. An expander is a list of sets of spatial nodes, where the number of node-sets in each expander's list gives the 'length' of the particle. With each iteration of the particle, each expander it owns is 'advanced'. We advance an expander by looking at all nodes in the graph that are neighbors to that expander, but which are not currently elements of it. These are then made members of a new set. That set becomes the topmost item in the expander's list. The previous topmost item is moved down the list. The bottom node set in the list is removed from the particle. (See fig 1.)

At the beginning of the experiment, the pseudo-particle is started at the graph's starting point with a single expander. That expander holds only the insertion node in its top set and empty

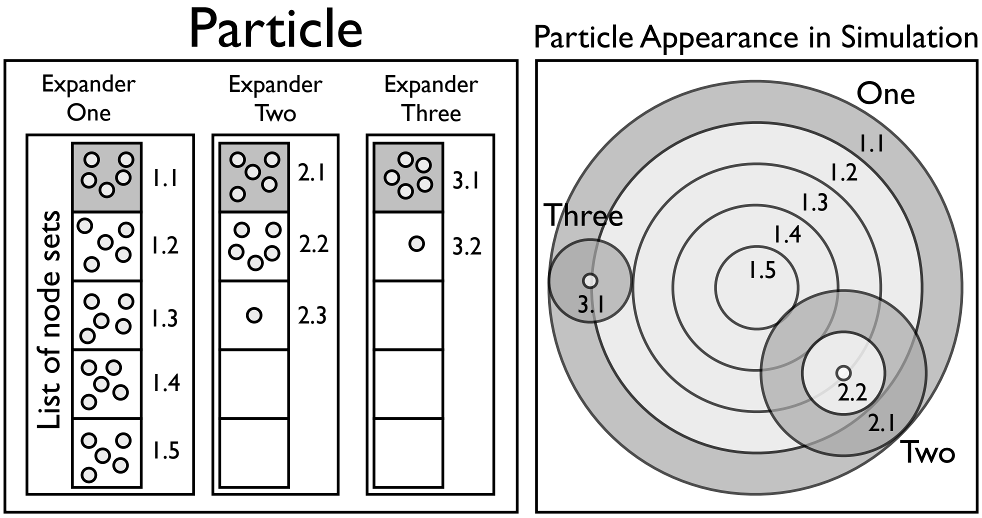

Figure 1: Pseudo-particle structure used to achieve self-interference in a simulated double-slit experiment. A particle is composed of a set of non-interacting excitation waves of fixed duration.

sets in the rest of its list. However, whenever a virtual event site is encountered by the particle, a new expander is added to the particle starting at that node. Thus, as the expanders grow, an ever increasing number of expanders are added to the particle. When viewed, the particle appears as a growing collection of ripples. In effect, the inclusion of event sites enables the particle to be represented by a set of non-interacting excitation waves.

When the particle reaches the far end of the graph, we employ a fixed probability of 'detection' every time an expander passes over a detector node. If a detection occurs, the spatial nodes referenced in the topmost sets of all expanders are gathered and scored based on the number of expanders they belong to. We then select one of these nodes at random as the location to which the pseudo-particle 'collapses'. If such a collapse point corresponds to a detector node, then a detection is recorded at that point. (Note that while this model uses a very simplified model of quantum mechanical wave collapse, there is nothing in its architecture that prevents us from extending it to model decoherence.)

Over a sequence of detections produced by many pseudo-particles, a smooth interference pattern slowly builds up across the detectors at the back of the graph. Changing the length of the pseudo-particle alters the distribution of peaks in the interference pattern, just as particle wavelength alters the distribution in a physical experiment. Placing a detector at one of the slits causes the particle to collapse at that slit with some finite probability, and the interference effect is lost. The results are obtained without the use of complex numbers, wave functions, or indeed any continuum mathematics at all.

Indeed, because a wave-function is not used, the wave-like behavior of the system does not degrade as the scale of the simulation is increased. Given sufficient memory, we can create simulations in which pseudo-particles travel for as far as we like without suffering some kind of degradation in wave-like behavior as we might expect with a simulation based on something like a cellular automaton. Also, because detection happens at a single node, particles appear point-like when interacted with, even though the structure that describes a particle can be composed of an arbitrary number of nodes.

One might argue that because we have employed random numbers in order to obtain this effect, we have not really gained anything over a traditional approach. However, numbers such as those required here can be trivially obtained via an algorithmic pseudo-random number generator.

This simulation succeeds because the notion of locality has been replaced by the notion of set membership. Set membership is used to model the associations between any two nodes, whether they are treated as particles, or part of the spatial background. In this approach, 'locality' is a consensus notion of distance that emerges from the bulk relations of many spatial nodes and which does not need to be conformed to by individual particles.

3.4 General Relativity

While research into simulations capturing the phenomena required by general relativity are still in early stages, some preliminary results have been achieved that are worth noting. Firstly, the pseudo-particle algorithm described in 3.1 is equally at home on distorted surfaces as it is on flat, Euclidean sheets. When given a graph derived from a curved manifold, the pseudo-particle follows curving, geodesic paths across it. Secondly, when the graph is kept spatially flat but is distorted in the time direction through the incorporation of stochastically distributed directed links, geodesics are also seen. Thirdly, when a helical, hierarchical particle of the sort described in 3.2 is applied to such a graph, distortions in that particle's subjective experience of time are witnessed while the geodesics are traversed.

Perhaps more importantly, some progress has been made in the self-organizing dynamics of spatial graphs. In order for any discrete model of the sort we have considered here to be considered viable, there must be a mechanism through which an approximation to smooth spatial geometry can be obtained from local, iterative rules. This is because constructing a spatial graph by pinning nodes onto a manifold and computing their distances requires exactly the kind of dependence on continuous structures we wish to avoid.

The simplest algorithm we have found so far that synthesizes geometry uses the random addition of nodes with fixed degree to a graph, followed by a process of pairwise arc removal. Combined with a mechanism for preferential attachment, it's possible to create relatively smooth, irregular surface graphs of any required dimension. Work in this area is ongoing.

4 Conclusion

While the work summarized here is speculative, and not the foundation for a unified physical theory, it illustrates is that many of the properties of physical systems that are assumed to require continuum mathematics can be modeled simply using iterative functions and directed graphs. Furthermore, there is no reason to believe that this approach cannot be extended to capture all remaining symmetries the same way. Refreshingly, reconciliation of quantum and relativistic effects appears to trivial in this paradigm, though difficulties lie elsewhere.

In order to achieve the results so far obtained, it has been necessary to revisit notions such as relativity and wave behavior in order to ask exactly what the requirements for matching experimental results actually are, rather than what it takes to match the descriptive tools we are used to using. Asking such fundamental questions is surely good for physics, regardless of what results are obtained.

We believe that there is a great deal to do in the domain of ‘computer-aided thought experiments’. While it may be some time before such experiments can be of practical use, it is our hope that the successes so far will stimulate further interest in discrete models, and encourage others to increase their scope. It is only by open-minded, courageous exploration that we will be able to identify the fundamental rules of Nature, regardless of what form they eventually turn out to take.

Video

- Motion in 2D: `http://www.youtube.com/watch?v=Y_yCxcjYPmo`
- Motion in 3D: `http://www.youtube.com/watch?v=3w4A6m26WI4`
- Geodesic motion: `http://www.youtube.com/watch?v=n3jnKejhX-Q`
- Relativistic motion: `http://www.youtube.com/watch?v=ggd8Z1fZwTA`

- Relativistic motion: `http://www.youtube.com/watch?v=ggd8Z1fZwTA`

References

[1] L. Floridi, *"Against Digital Ontology", Synthese, 168.1, 151-178* (2009).

[2] M. Tegmark, *"The Mathematical Universe", Foundations of Physics 38 (2): 101150* (2008).

[3] J. Schmidhüber *"Algorithmic Theories of Everything", Technical Report IDSIA-20-00, Version 2.0; 20* (Dec 2000)

[4] A. Lamb, *"A Glider for Every Graph: Exploring the Algorithmic Requirements for Rotationally-Invariant, Straight-Line Motion," Complex Systems vol. 18 (4)* (2010)

[5] A. Lamb, *"Dense Graphs, Node Sets, and Riders: Towards A Foundation for Particle Physics without Continuum Mathematics," Complex Systems vol. 19 (2)* (2010)

[6] S. Aaronson, *Book Review: A New Kind of Science, Quantum Information and Computation 2(5):410-423* (2002)