

What is ultimately possible in physics?

1. Abstract

Available written records suggest many human cultures hold or have held that there is more to reality than the observable physical Universe. Modern physics conforms to this pattern, postulating an invisible 'wave function' to explain observable phenomena.

Many modern cosmologists appear to believe that the initial state of the universe was constrained in a way that dictated its evolution to its present state, which includes us.

The Western Judaeo-Christian tradition attributes this constraint to an omnipotent, omnipresent, omniscient being called God. God is believed to have eternally pre-existed the Universe and to be in essence completely distinct from and unlike the Universe. The standard Western theological model proposes that the only constraint on God is self consistency (1) .

Here we explore the hypothesis that God and the Universe are the same reality, leading to the conclusion that ultimately physics and theology have the same subject and that the Universe is subject to no externally imposed constraint. The world of our experience is constrained only by self consistency as traditionally attributed to God.

We explore this constraint in terms of a logical model based on the extension of practical finite computer networks into the transfinite domain first explored by Cantor and applied by to the foundations of quantum mechanics by von Neumann using the function theory developed by Hilbert (2).

2. Historical models of God and the Universe

Scientific conjectures are generally expressed as formal models which can be represented by invariant written symbols. Here we are concerned with the reconciliation of three models: the model of God initiated in antiquity and recorded in its completed form by the thirteenth century theologian Thomas Aquinas; the general theory of relativity developed by Einstein; and quantum theory.

Aquinas produced five famous proofs for the existence of God. These proofs all take the same form: the Universe as we understand it cannot explain itself. Since it clearly exists and operates in an intelligible manner, we must postulate some entity other than the Universe to explain it; this entity we call God. The key point, that the universe cannot explain itself, derives from the model

of the Universe that Aquinas inherited from Aristotle (3).

The general theory of relativity takes the form of a differential equation (the Einstein equation) which governs the large scale evolution of space-time (4). Although Einstein sought a static solution to this equation, it has since been shown that no such solution exists and that the space-time must either be expanding or contracting. Observational cosmology shows that it is expanding and mathematical investigations of Einstein's equation show that it implies the existence of an *initial singularity*, at which point the universe has no size or structure (5).

Formally, we may identify this initial singularity with the traditional model of God, both of which may be seen as without structure and as the source of the current universe. The only difference is that rather than seeing the Universe as having been created 'outside' God, as in the traditional view, we consider the possibility that Universe we experience is 'inside' or identical with God.

The mathematical formalism of quantum mechanics assumes that states of the physical Universe can be represented by *state vectors* in complex Hilbert space of finite or infinite dimension. The joint state of two communicating quantum systems is represented by vectors in the tensor product space of the Hilbert spaces of the constituent systems.

The continuous evolution of state vectors in an isolated quantum system is described by unitary operators on their Hilbert space governed by Schroedinger's equation. Since such a system is isolated, however, this continuous evolution is not directly observed but is inferred from its consequences.

The continuous evolution of an isolated quantum system is held to be interrupted by an observation or measurement. An observation is modelled as the interaction of one quantum system with another, and takes place in a state space which is the tensor product of the state spaces of the individual systems (6). Since such observations take place within the universe, they may be seen as increasing the complexity of the Universe, and so provide a mechanism for the differentiation of the initial singularity into the universe we currently observe.

This mechanism of complexification has its analogue in ancient theology. Christianity added the notion of the Trinity to Hebrew monotheism. Augustine and Aquinas, seeking to explain this development, produced a model of the Trinity based on their understanding of human psychology (7). God the Father, observing himself, produced the Word or Son of God. The love of the Father for the Son is realized as Holy Spirit, the third Person of the Trinity. We follow this analogy, but do not cut off the complexification of the initial singularity at three sources, seeing it rather as differentiating without end, like the transfinite numbers, every quantum interaction adding to the complexity of the overall system.

3. Quantum mechanics and communication networks

Quantum mechanics enjoys a long established reputation for counterintuitivity. For many, schooled in the classical Newtonian physics of massive particles moving in continuous space, it is simply a very effective algorithm for predicting the probability of certain events. Seen as describing networks of communication, however, quantum mechanics acquires clear meaning, since we are natural network communicators.

First: the mathematical theory of communication founded by Shannon begins with a source which is defined by an alphabet of letters (corresponding to quantum mechanical eigenvalues) each of which has a certain probability of emission (8). The sum of these probabilities is normalized to 1. Like a communication source, quantum sources are also normalized, so that the sum of the probabilities of distinct eigenvalues is one. The normalization of quantum sources is reflected in the unitarity of the operators which govern the evolution of a quantum system.

Second: although the evolution of quantum states in an isolated quantum system is considered to be continuous, every observation of such a system yields a discrete eigenvalue corresponding to an eigenfunction of the system evolution operator. The mathematical theory of communication shows us that to defeat error in a noisy channel, messages must be encoded in blocks designed to maximise their distance apart in communication space so minimizing their probability of confusion. This is equivalent to quantization or digitization. A message can be transmitted without error when there is no overlap or confusion between it and other messages, that is if it is orthogonal to other possible messages. Distinguishable quantum states are also orthogonal to one another.

Third: the continuous unitary evolution of quantum states is governed by a differential equation (Schroedinger's equation) which has as many solutions as the dimension of the relevant Hilbert space. The observation of such a system is held to cause a mysterious, discontinuous 'collapse of the wave function', yielding an eigenvalue corresponding to just one of the possible eigenfunctions. From a network point of view, this corresponds to the reception of just one of the many messages stored at different points in the network, just as this sentence corresponds to just one of the many sentences stored in the neural network that forms the physical basis of my mind.

Fourth: Einstein and others have considered quantum mechanics incomplete because it does not provide an exact prediction of the outcome of a particular set of initial conditions, but only the probability of certain possible outcomes corresponding to eigenfunctions of the observation

operator. The mathematical theory of communication takes no account of the meaning of the messages that it treats, but is based solely on the statistical structure of its sources. A consequence of communication theory is that a coding system operating at maximum efficiency produces messages that are indistinguishable from random noise. Only a receiver in possession of the inverse of the encoding algorithm can decode such a message.

Fifth: a further indication that it is plausible to consider the Universe as a communication network is provided by the structure of space-time revealed by the special theory of relativity. Events at rest in any local inertial frame are time ordered, and we believe that an event can only influence events that follow rather than precede it in time. The velocity of light is taken as the fixed and finite maximum velocity for the transmission of information from one point in space-time to another. One consequence of the finite maximum velocity of communication is that events in an inertial frame moving relative to oneself look different. Assuming that the laws of physics are the same in every inertial frame, we may derive velocity dependent Lorentz transformations which bring the distorted appearance of moving systems back to the normal appearance of things in our own rest frame. There is nothing special about the velocity of light in this derivation. The same structures are found when we substitute the speed of the postal service, for instance, into the causal ordering of human events. The algorithms used to defeat error in communication induce delay. In order to encode a message of n symbols into a transmissible signal function, the encoding machine must wait for the source to emit n symbols, even if the computation required for encoding is instantaneous. Since computation itself involves error free communication within the computer, we can expect it too to add delay. We are therefore led to believe that communication delay, quantization and error prevention are connected. Further, since the least delay corresponds to the maximum velocity, we propose that the algorithm used to encode information into signals that travel at the velocity of light is the fastest algorithm in the Universe.

Sixth: quantum mechanical systems exhibit entanglement. Quantum systems that have been in contact with one another (like electrons in a singlet state) maintain a non-classical correlation when they are separated. This phenomenon first came to notice through the Einstein, Podolsky Rosen (EPR) thought experiment, since frequently realized. EPR proposed that entangled particles could act upon one another at a distance even though their separation was spacelike, requiring something greater than the velocity of light to account for their correlation (if it is due to communication): 'spooky action at a distance' (9) (10). The observed correlations between entangled photons have been found to propagate at many times c , and the measurements are compatible with infinite velocity, ie instantaneous transmission, over many kilometres (11). The

network picture may account for this phenomenon. Error resistant encoding introduces delay. In practical communication networks a message originating with user *A* is passed down through the software layers in *A*'s computer to the physical layer which carries it to *B*'s machine. It is then passed up through *B*'s software layers until reaches a form that *A*'s peer *B* can read. By analogy, communication between one system in the Universe must pass down to the ultimate physical layer (which we might identify with the structureless initial singularity) then up again to the peer system receiving the message. It may be that the simplicity (high symmetry) in the lower layers of this network make encoding for error prevention unnecessary, so that instantaneous communication is possible.

Seventh: the union of quantum mechanics and the special theory of relativity yields quantum fields theory, currently our best description of the physical Universe (12). As suggested above, we might attribute the finite velocity of light to the delay introduced by error resistant encoding. A key idea in quantum field theory is local gauge invariance which tells us that there is a set of degrees of freedom associated with every point in space-time whose values are independent of their values at neighbouring points. Local gauge theories have nothing to say about these values, and so are consistent with all of them. This feature of these theories is exactly what we expect in a compliant communication network, that is one indifferent to the actual content of the messages transmitted. This same property is manifest in the mathematical theory of communication which is indifferent to actual messages and their meaning, and is interested only in their statistical structure.

Finally: although the mathematical foundations of quantum field theory are somewhat shaky, the results obtained by applying Feynman's diagrammatic method are very good, particularly in quantum electrodynamics (13). We may consider a Feynman diagram as a picture of a fragment of the Universal network. As we know, the size of the diagram increases with the order of perturbation considered. From a quantum mechanical point of view, the strength of a perturbation is proportional to its energy, that is its rate of communication. To obtain a perfect picture of any single event in the universe, we have to consider the diagram which includes all events in the past light cone of that event, that is all messages in the Universal network relevant to the event in question, beginning from the initial singularity.

4. Turing machines and '*logical continuity*'

Given that the Universe described by quantum theory is a computer communication network, we can say something to the question What is ultimately possible in physics? A computer network is simply a set of communicating computers. Practical networks are layered. Since all information

is represented physically, the foundation of any network is the physical layer. Built on this layer are higher layers of software which handle error prevention and correction, the routing of messages and the transformation of messages between machine to human readable forms. All these higher layers are executed digitally, usually using binary digits.

Physics generally assumes that the universe is geometrically continuous, a natural assumption, given the continuous appearance of macroscopic spatial motion. On the other hand all communications, observations and discussions in the Universe are quantized. Experimental physics revolves around classifying ('binning') and counting events. When we observe the output of physicists and mathematicians ('the literature') we see that it too is quantized, into discrete volumes, articles, words and symbols, like this.

Continuity is a very strong constraint on the variety of functions and models. The total number of functions on a domain of cardinal n is the cardinal of the set of permutations on a set of n elements, i.e. $n!$. Of these, only $2n$ are continuous in the sense that each element is replaced by its nearest neighbour to realize a cyclic subgroup of the group of permutations. The ratio of continuous functions to all possible functions is thus $2n/n!$.

Continua are essentially unobservable, since there are no 'marks' to observe. So we cannot observe the continuous unitary evolution of an isolated quantum system. When we observe such a system, we do not see the whole continuous system, but only one or other of the basis states (eigenvectors) of the operator we use to observe the system.

This suggests further that the correct understanding of continuity in the universe is not 'geometric continuity' but 'logical continuity'. The archetype of logical continuity, corresponding to a continuous function in analysis, is a proof: a logically watertight connection between some hypothesis (say Euclid's axioms) and some conclusion (eg Pythagoras' theorem) that can be executed by a suitably programmed deterministic computer (14). A halting Turing machine is a logical continuum or proof, moving deterministically from an initial state to a final state.

From a practical point of view, logical continuity takes precedence over geometric continuity. So our practical mathematical understanding of continuity is based on logical processes (like the *epsilon-delta* argument) that show that a function is continuous if the distance between points in its range tend to zero as the distance between corresponding points in its domain tend to zero (15).

Geometrical *continuity by proximity* motivated Cantor's search for the cardinal of the continuum (16). Using logical continuity, Cohen has shown that Cantor's approach, using theory based on sets of discrete elements, can say nothing about classical geometrical continuity (17). Cohen's

logically continuous argument shows that Cantor's continuum hypothesis is independent of set theory.

Nevertheless, continuity, considered as a symmetry, is a very productive idea in physics. From a network point of view, logical and geometric continuity correspond at the ultimate physical layer which in turn corresponds to the initial singularity described by the continuous unitary evolution of quantum mechanics. At this level the entropy (and information content) of the universe is zero, since there are no distinguishable or countable symbols to account for any higher entropy. The entropy of the universe is increased by the differentiation introduced by communication. We may imagine a formal correspondence between each layer of the universal network and each transfinite cardinal in the Cantor universe. The hierarchy of transfinite cardinals thus gives us an absolute scale of entropy in the Universe.

5. What is the ultimate size of the universe?

Computers implement the algorithms of formal logic, that is the manipulation of symbols and strings of symbols according to certain rules. Peano's axioms show us how to create a countable infinity of symbols, the natural numbers. Goedel and Turing have shown us how to use strings of such symbols to represent algorithms. Cantor has shown us that the natural numbers are just the beginning of a transfinite hierarchy of symbols. The set of permutations of the aleph(0) natural numbers give us a set of aleph(1) complex symbols, whose permutations give rise to a set of aleph(2) more complex symbols, and so on without end. We can imagine the permutations in this system being executed by Turing machines, which are themselves represented by strings of symbols.

How big is the universe? This depends on how we measure it: physically, in terms of the cardinals numbers mass, length and time, it is already huge. Logically, it is much larger, corresponding to the transfinite ordinal numbers. Let us assume that the logical execution of a Turing machine corresponds to a quantum of action. Energy then corresponds to the rate of execution of computations. Let us further assume that each complete computation corresponds to the transmission of a message in the universal computer network. and that the transfinite Cantor universe serves as a 'phase space' in which the universe maps itself onto itself by computational transformations.

From a quantum mechanical point of view, every electron is identical, just as every execution of a NAND gate in a logic processor is a formally identical instance of a propositional function. When the overall context of the universe (or computer) is considered, however, every act is

different, identified by a space-time address. One bit in a computation may be a flag whose meaning is 'launch an all out nuclear attack' whereas another might be the least significant bit of a pixel in a featureless image. By differentiating every act in the Universe, the logical computer network picture reveals a universe transfinitely larger than that represented by the cardinal numbers we attribute to mass, length and time. This is consistent with the quantum no cloning theorem. While quantum mechanics predicts the frequency of messages on various channels, the network view of the universe sees every event as a message meaningful to some user of the network. The ultimate user of the Universal network is the Universe itself.

6. Quantum computation

Many hope that quantum (analogue) computers will turn out to be more powerful than Turing machines for two reasons. First, in quantum theory an operator like the Hamiltonian in Schrodinger's equation acts formally on all the elements of a superposition at once. This gives hope for the execution of massively parallel calculations in one operation. Second, because a continuous variable is held to be able to encode an infinite amount of information and quantum amplitudes are represented by continuous functions, it is hoped that quantum computers will effectively operate on 'words' of infinite length.

To the contrary, we have the approach taken by *algorithmic information theory*, which measures the total information in a transformation by the length of the program required to implement the transformation (18). From this point of view, the equation $x = y$ defined on the real line does not contain an infinite amount of information, but only the few bits represented by the symbolic expression ' $x = y$ '. This alerts us to the fact that the entropy we assign to a set depends on how we decide to count its elements. From a computational point of view, the algorithmic measure seems most appropriate.

7. Evolution

From a logical point of view, we are not so much concerned with the geometric size as with the algorithmic complexity of the Universe. Although there are only a countably infinite number of different Turing machines (computable functions), the symmetric network described above is transfinitely larger, so that we may see computation as a limiting resource in the Universe. This lays the foundation for evolution: many possible processes, confronting limited resources, select for those processes that use the available resources most efficiently to secure their own survival and replication. This process of selection may be reflected in the utility of the principle of least

(or extremal) action in searching for candidate models of the Universe (19).

8. A final objection

We have described a formal model of the Universe based on the Cantor universe and Turing machines, a system which approaches the limits of consistent mathematical complexity. We are proposing that this serves as both a model of God and the Universe, thus formally unifying the two entities. It may be objected, however, that while our model approaches the limits of complexity and seems to fit the extremely complex Universe in which we live, the classical God is completely simple, *omnino simplex*, so that our hypothesis must fail.

The answer to this problem lies, I feel, in the realm of topological fixed point theorems, pioneered by Brouwer (20). Fixed point theorems show that when a bounded set is mapped onto itself, some points must remain fixed. We may consider each layer in the universal network as bounded by the layer above it. Each of us is a peer in a human layer of the Universal network, and our interactions through communication with one another may be considered as mapping of humanity onto humanity. Each of us, during our lifetime, serves as a fixed point in this layer, created at birth and annihilated at death. We can thus reconcile a completely dynamic system (Aquinas: *actus purus*) with apparently static formal elements which appear to be outside time and dynamics. From this point of view, each of us is an observable message or particle in the Universal network. If we abstract from complexity, we are isomorphic to the particles of quantum field theory, subject to creation and annihilation within the overall system just as they are.

9. Conclusion

The ideas presented here are the tip of a very large iceberg. We can for instance distinguish past from future by considering the past as the fixed output of halted computations, whereas the future remains to be determined. Further, since Turing has demonstrated the existence of incomputable functions, there are dynamic processes that will never end. Part of the counterintuitive difficulty of quantum theory lies in its reliance on complex numbers. This problem may be resolved in the network model by noting that the definitive feature of both complex numbers and computations is their cyclical or recursive nature. The only process that is not repeated is the total execution of the universe itself.

The principal targets of the project from which this work derives, however, are theology, religion and global peace, both within the human species and between humanity and the rest of the

world. Theology, as it exists today, is largely based on ancient texts written long before the development of modern theories of our place in the world.

By developing a theological model with observable consequences, we can, in the long term, expect a unification of theology analogous to the unification of the other empirical sciences that we have seen in the last few centuries. We all live in the same Universe, so if our science accurately tracks the universe, we will all end up at the same conclusion. The unification of theology, might, in turn, promote the unification of religion and the mitigation of religious conflict. Further, a theological model which places humanity firmly in the Universal context might provide us with a global spiritual outlook able to hasten the technological and behavioural developments necessary to bring our species into harmony with the rest of the world. We believe that the sun will underwrite life for the next five billion years. If we play our cards right, the human career is just beginning (21).

10. References

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