Mission Impossible

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Abstract

The laws of physics dictate what is physically impossible. But physical laws are often superseded by newer, more permissive laws. Therefore only the final theory of physics can determine what is and is not ultimately possible. I examine some of the fundamental tenets of quantum field theory and general relativity and argue that it is not certain that any of our currently sacred principles will survive unscathed in the ultimate theory of physics.

Progress Unbounded?

The breath-taking advance of technology is best summed up by Moore's law. Back in 1965 Gordon Moore, co-founder of Intel, made the brave prediction that computer complexity, which is basically computing power, would double every 18 months. Exponential growth being what it is, this means that every fifteen years computers would become a thousand times more powerful. Indeed, today even motorcycles have chips that are far more sophisticated than the computers that guided the first spacecraft to the moon. In the meantime, the human versus computer chess showdown has turned into a walkover for the computer. And we haven't even started to tap the vast potential of quantum computing.

Amazing as this is, technologically we are merely scratching the surface of what's physically possible. In 1959, Richard Feynman gave a brilliantly prescient after-dinner speech, which was later published with the title "There's Plenty of Room at the Bottom." Feynman realized that, because we do not encounter new laws of physics until we get to smaller than the atomic scale, we could in principle control materials down to the size of the very atoms.

This ability to manipulate atoms individually — an ability that is already emerging in labs — will lead to staggering advances. For example, both diamonds and graphite are made of carbon atoms; the vast difference between these two materials comes from how the carbon atoms are arranged. By rearranging atoms, it is not impossible to turn pencil lead into diamond. And recently, new configurations of carbon atoms called fullerenes have been discovered which have fantastic properties: they are ultra-light and a thousand times stronger than steel. There is now serious talk of using filaments of fullerenes to create a cable for a space elevator. Future satellites, rather than being launched by rocket, might simply ascend this elevator, a stairway to heaven, climbing up the cable like Jack on his beanstalk.

As Feynman understood, it is through knowing the laws of physics that we can say what is and is not possible. Here's what's possible. It's possible to fly (with some discomfort) from San Francisco to the Azores in under 20 minutes. It's possible to create minuscule robots that could fix our bodies from the inside without the need for an operation. It's possible to use black holes as power sources. No scientific laws would be violated if such things happen. If they don't happen, it will be because they're either technically difficult or terribly expensive — but not because they're impossible.

A Brief History of the Impossible

> Why, sometimes I've believed as many as six impossible things before breakfast.
>
> — The Queen in Lewis Carroll's *Through the Looking-Glass*

From a scientific point of view, then, it is perhaps more interesting to ask not what is possible, but what is impossible. We regard certain things as impossible not so much because we've tried them and failed, but because the equations of our best scientific theories forbid them. At first sight, it would seem easy to state what is impossible: all we need to do is consult the laws of physics. Superluminal motion, perpetual motion machines, causality-violating time machines — surely these are illegal. By discovering scientific laws, we set boundaries on what even an infinitely advanced civilization would be able to do; we are all law-abiding citizens when it comes to the laws of science.

However, a look at the fate of the physical laws of the past provides a cautionary tale. Scientific laws, while written in the language of mathematics, are not the same as mathematical laws. In mathematics, it is utterly impossible for two plus two to equal five. But scientific laws are only as good as their assumptions. And as we have shed our assumptions, our laws have become more general but also more liberal.

Take alchemy. In the days before chemistry, before atomic elements were known, alchemists — medieval chemists — tried hard to turn one substance into another. Even the great Isaac Newton spent some thirty years of his life persistently but fruitlessly trying to turn lead and other metals into gold. Not knowing chemistry, poor Newton did not know that in all chemical reactions the basic elements are unaltered; neither the atomic number, Z, nor the atomic mass number, A, could vary. Like reassembled Lego pieces, atomic elements could be combined as different compounds but the building blocks — the elements themselves — do not change. By the laws of chemistry, the transmutation of one element into another — the holy grail of alchemy — is simply impossible.

Impossible? Well, no. The discovery of radioactivity showed that it is, after all, possible for one element to turn into another. So we see that as scientific knowledge broadened from chemical to nuclear processes, what had been considered impossible became possible. The lesson here is not that we should abandon chemistry, but that scientific theories can be limited in scope. Laws have their jurisdiction.

Similarly, mass was thought to be conserved. Indeed, the very first section in Landau and Lifshitz' book on *Fluid Mechanics* presents the continuity equation that arises from the conservation of mass in any fluid volume. What could be more basic? And yet, with $E = mc^2$ this simple law was found to be valid only under the restricted circumstance of non-relativistic physics.

The history of particle physics in particular is replete with violations of the previously inviolable. Parity was thought to be conserved until the decay of Cobalt-60 by beta decay was studied. In the Standard Model of particle physics, the conservation of baryon number (as well as lepton number) is another law that was regarded as inviolable by a generation of particle physicists. But Grand Unified Theories that supersede the Standard Model predict proton decays of the form

$$p^+ \rightarrow e^+ + \pi^0 \tag{1}$$

One could go on. It was thought that bosons and fermions were distinct particles that could never be combined into a common multiplet. And yet, when supersymmetry was theoretically discovered, the Coleman-Mandula theorem was bypassed.

What these and other examples reveal is that, as the scope of science increases, the illegal can become legal as we discover loop-holes in earlier, more limited laws. Indeed, it has become almost an article of faith that no-go theorems can be circumvented. Thus we find that identifying the truly impossible is much harder than we might have thought. To really know what laws are inviolable, we must have the ultimate laws. What then is truly impossible? The ultimate laws of science will be determined by a theory of fundamental physics. But without a final theory of physics in hand, it is hard to say which of the present scientific laws will survive.

To be clear, we should specify that when we speak of which laws will survive, we don't of course mean which will survive as *ideas*. As ideas, gauge symmetry or locality or any of the other broad principles that we have are both sound and profound. What is at issue is whether they always apply. In other words, we are interested in finding out what laws hold under all circumstances. Such laws can be used to determine what is truly impossible.

First, however we must draw a distinction between those laws that tell us about kinematics or dynamics and those that are statistical in nature.

Improbable versus Impossible

In *The Hitchhiker's Guide To The Galaxy*, Zaphod Beeblebrox, fugitive president of the galaxy, steals a spaceship powered by an Improbability Drive. Whenever the Improbability Drive is switched on, it makes highly improbable things happen to the ship: when two nuclear-armed guided missiles approach, they are spontaneously transformed into "a bowl of petunias and a very surprised-looking whale."

The notion of improbability is closely connected to entropy and the second law of thermodynamics. It was Ludwig Boltzmann who realized that the extraordinary diversity of time-asymmetric phenomena could be encapsulated by a single potent equation:

$$\frac{dS}{dt} \geq 0 \, . \tag{2}$$

In words: entropy increases with time. Here entropy can be defined either classically, as the logarithm of the volume (Γ) of phase space, or quantum-mechanically, from the reduced density matrix (ρ) or as the logarithm of the number ($\mathcal{W}$) of microstates in a given macrostate:

$$S = k_B \ln \Gamma = -k_B \mathrm{Tr} \rho \ln \rho = k_B \ln \mathcal{W} \; , \tag{3}$$

the last of which is the epitaph etched on Boltzmann's gravestone. Boltzmann's crisp statement of entropy increase supplies the thermodynamic arrow of time: given two snapshots of a system (more precisely, of its phase space) at different instances, we say that the one with the greater entropy comes later. Indeed, it provides a rule for when a thermodynamic arrow of time exists at all:

$$S_{\mathrm{actual}} < S_{\mathrm{maximum\,possible}} \; . \tag{4}$$

Whenever the actual entropy is strictly less than the maximum entropy, the system will evolve towards maximal entropy. Things will change in time, becoming more disordered; an arrow of time is induced. Conversely, when the system has already reached maximal entropy, nothing much happens and there is no arrow of time at all.

The second law of thermodynamics, rooted as it is in statistical mechanics rather than in any particular kind of dynamics, has sweeping, awesome power. Rather than separate laws explaining why glass shatters but never unshatters, or why eggs break but never unbreak, or why black holes decay but radiation doesn't collapse, we have one all-encompassing law. One merely has to compare entropies to find the correct sequence of events. The low entropy state (wine glass, intact egg, black hole) comes before the high entropy state (broken shards, eggy mess, Hawking radiation). Indeed, Einstein regarded the second law of thermodynamics as the physical law that was least likely to be replaced: "It is the only physical theory of universal content which, I am convinced, that within the framework of applicability of its basic concepts will never be overthrown." Thus we might perhaps be able to file "perpetual motion machines of the second kind" under what is truly impossible.

However, the second law of thermodynamics is only true on average. Small deviations from equilibrium can and do happen, and Boltzmann was able to quantify the likelihoods of such fluctuations and the time-scales over which they could be expected. In essence, the probability of such a fluctuation decreases exponentially with its departure from maximal entropy:

$$P \sim \exp(\Delta S) \; . \tag{5}$$

The utility of such considerations can be illustrated by one example. Take protein folding. It had been thought by some biologists that proteins fold into their chosen conformation (shape) purely by chance. Yet entropy counting tells us that this cannot be true: the time taken for a protein to find its correct conformation would exceed the lifetime of the universe, an observation sometimes known as the Levinthal paradox. We therefore conclude that the process by which proteins fold must be directed, not random.

In any case, for determining what is technologically impossible, the merely improbable may not be enough. There are lesser versions of Improbability Drives that we could come up with. After all, quantum-mechanical tunneling is quite unlikely and yet we are able to exploit it to create scanning tunneling microscopes. Indeed, in quantum mechanics, as Feynman famously said, what is not expressly forbidden is mandatory. So even if something is unlikely, so long as it is not expressly forbidden, it will happen. We must therefore determine not just what is improbable, but what is impossible. For that we have to turn to the ultimate dynamical laws of physics.

Do We Know Any Of The Ultimate Laws?

The edifice of theoretical physics today rests on two pillars: the Standard Model of particle physics and Einstein's general theory of relativity. (We do not include string theory here mainly because its central principles have not yet been elucidated.) As a quantum field theory, the core principles of the Standard Model are locality and symmetry, particularly Lorentz symmetry and gauge invariance, and of course the idea that the basic degrees of freedom are operator-valued functions of spacetime. The core principles of general relativity are the equivalence principle and diffeomorphism invariance.

Which of these principles are likely to hold under all circumstances? This is really a question about the principles that will continue to hold in a quantum theory of gravity. We can discard the equivalence principle; even the generic low-energy effective action of string theory contains a dilaton which violates the equivalence principle. Thus the equivalence of inertial mass and gravitational mass is not one we expect in general to hold.

The breakdown of locality in quantum gravity is easy to see. Quantum mechanics states that

$$\Delta x \; \Delta p \geq \frac{\hbar}{2} \tag{6}$$

Thus Δx can be made as small as we like simply by increasing the spread in momentum. So high energies allow us to probe ever-shorter distances. However, this reasoning does not take into account gravity. Gravity does not allow for arbitrarily high-energy probes of short distances: eventually black holes form. Thus spatial resolution at an energy scale M cannot be finer than the Schwarzschild radius:

$$\Delta x \geq \frac{2GM}{c^2} \tag{7}$$

Combining the uncertainty principle with this restriction, we find (with $M \sim \Delta p$) that

$$\Delta x \geq \frac{\hbar}{2\Delta p} + \frac{2G\Delta p}{c^2} \tag{8}$$

and thus there is a shortest length scale which, rather unsurprisingly, is the Planck length. Clearly, locality, in the continuum limit in which it is used in quantum field theory, is a convenient fiction. String theory provides further evidence for the breakdown of locality since it contains extended objects like strings and branes which do not experience a point-like geometry.

But in fact locality may break down much earlier than the Planck or string scale. Strong evidence for the breakdown of locality comes from the study of black holes. The Bekenstein-Hawking entropy formula for black holes is

$$S_{\mathrm{BH}} = \frac{A}{4G\hbar} \tag{9}$$

The interpretation of this entropy is a contentious subject. That it scales as an area may indicate that, rather than enumerating the fundamental degrees of freedom of the black hole, the Bekenstein-Hawking entropy may actually be the entropy of entanglement. Entanglement entropy is the entropy that comes about when we neglect or do not have access to some subsystem. If Bekenstein-Hawking entropy is entanglement entropy, it would present no difficulties to quantum field theory. Indeed, it would be consistent with locality that only the degrees of freedom near the surface of the black hole would be entangled with the outside, thus yielding an entropy that scales as the area of the event horizon. However, there appears to be a growing consensus that this entropy is not entanglement entropy and that, like entropy in general, it encodes the number, N, of quantum-mechanical microstates of a black hole:

$$S = \log N \tag{10}$$

If this is so, then we must confront a shocking fact: the number of degrees of freedom in quantum gravity would scale as the area, rather than extensively, as the volume. With such scaling, locality would obviously be only an approximation.

Without locality, and with spacetime stripped of its fundamental status, Lorentz symmetry is likely to be in trouble. (Perhaps by exploiting this, a sufficiently advanced civilization would be able to send superluminal signals.) What finally of local symmetries like gauge invariance or diffeomorphism invariance? The conservation of mass in pre-relativistic physics and the conservation of baryon number in grand unified theories are both examples of global conserved charges. Their violations are allowed. By contrast, gauge charges, which generate gauge symmetries, must be conserved in quantum field theory. Will the conservation of electromagnetic charge therefore be a part of the final theory? Since gauge symmetries are merely redundancies, it would seem difficult to violate them. Nevertheless, since gauge symmetries are local symmetries and since locality itself is in doubt, it does not appear certain that we can rely on gauge symmetries as being part of the ultimate theory that describes our world.

We have seen that we cannot be sure of any of the current tenets of fundamental physics. It is possible that there are other principles — perhaps causality — that may survive in the final theory. Perhaps we already have some of these laws, but we do not know that with any certainty. It appears then that to find out what is truly not possible, we will have to continue our quest for a final all-encompassing theory.

Until then, we can still dream of the impossible.