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## ON EXPLAINING EXISTENCE

ABSTRACT: What are the limits of physics' explanatory power? Can physics explain everything? In this paper I discuss a somewhat broader question: can physics explain existence itself? I argue that genuinely ultimate explanations—those that really explain everything—involve the most basic and most general elements of logic. Such explanations cannot be done within physics unless physics undergoes a methodology shift more closely aligning itself with mathematics and logic. However, I give reasons for thinking that just such a shift might be in operation.

Even a solipsist agrees there is at least one thing!  
Roy Sorensen

### 1

#### WHAT PHYSICS CAN AND CAN'T DO

Physics is very good at providing explanations for physical phenomena. If we are puzzled, for example, by the appearance of a rainbow, then, with our physics hats on, we seek to provide an explanation of it, such that the phenomenon (or, rather, some representation of it) is derived as the conclusion of an argument involving some physical laws and some initial/boundary conditions. Understanding comes as a natural consequences of explanation too: if the rainbow *had* to be, given the laws and the conditions, then we can claim to have understood the phenomenon of the rainbow. We can, of course, also make predictions and exert some control over the natural world along these same lines.

However, what if the phenomenon we are puzzled about is the Universe as a whole (be it an ensemble or a unique instance) and the problem of where it came from? Cosmology and cosmogenesis were, until relatively recently, thought to be outside the 'proper' domain of science, to be relegated instead to the armchair speculations of metaphysicians and theologians. Of course, theory and observation have advanced to the stage where we might expect scientific explanations (and so understanding) of these deep questions too. We have theories of the structure and origins of the universe that submit themselves to elements of the standard scientific method.

But now what if we are puzzled about existence itself? Never mind explaining why the universe has the contents it has, distributed as they are, with their properties 'just so', and so on. Why is there *anything at all*? This is really *the* ultimate question: why is there something when, presumably, there might not have been? There are two ways to go about answering this kind of question:

1. Show that existence is in fact *necessary*, so that there couldn't possibly have been nothing—in other words, we deny that there might have been nothing, so that the presumption above is seen to be mistaken.

2. We somehow give a ‘bootstrap’ explanation showing how consistency alone brings reality into being, perhaps via some kind of (non-vicious) explanatory loop.

These two strategies really tackle slightly different questions, and that it is only the former that is capable of fully explaining existence *simpliciter*, of explaining why there is something rather than nothing. More explicitly: the only way one can explain existence (not the *form* of existence, but that there is existence at all) is to demonstrate that non-existence is a logical impossibility. Any contingency that enters in to an explanation will leave open a logical gap that renders the explanation incomplete—the second strategy fails in just this respect for there is no reason given for the existence of the consistency loop to begin with.<sup>1</sup> Thus, although one can answer the question ‘why this Universe?’ using consistency principles, one cannot answer the more general question ‘why is there being?’

It has usually been supposed that physics cannot answer this kind of deep question, but is this true? Or in other words, are strategies of the kind mentioned above out of bounds for physics? It is certainly true that physics, in the past few centuries at least, has tended to tackle experimentally tractable problems; but theory has overtaken experiment in very recent years and one tends to be guided exactly by constraints imposed from ‘internal’ demands, such as mathematical consistency.

Moreover, recent developments in physics have changed the meaning of such questions. When we ask “why does something exist (rather than nothing)?”, we are relying on a tacit assumption about what that ‘something’ is. Is it a nice single compact universe, or is it a vast *ensemble* of universes? What kind of ensemble? A restricted one or an unrestricted one? Where do the restrictions come from? From physical plausibility or from mathematical consistency? The former would seem to be too limited to explain everything, for physical plausibility is surely too subjective: what is implausible at one time has later become the very embodiment of plausibility, even providing the very yardstick of future decisions about plausibility. I will argue that physics *is* capable of answering even this, the very deepest of questions, providing certain (not unreasonable and unnatural) methodological shifts are undergone. Furthermore, just such a shift appears to be happening in present fundamental physics.

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<sup>1</sup>Here I have in mind schemes like Wheeler’s ‘meaning circuit’ whereby the form the universe takes on—replete with its laws and constants—is due to decisions made by observers: the observers postselect the earlier physics which brings about their own existence (see [17]). “Minuscule though the part is today that such acts of observer-participancy play in the scheme of things, there are billions of years to come. There are billions upon billions of living places yet to be inhabited. The coming explosion of life opens the door to an all-encompassing role for observer-participancy: to build, in time to come, no minor part of what we call its pastour past, present and future but this whole vast world.” Cf., also, Hawking and Hertog [10] for a similar ‘top-down’ approach to cosmology.

### WHY QUESTIONS

There are a variety of interesting and relevant why-questions (concerning supposedly contingently existing things) that lead us to the grander question (why something rather than nothing?) posed above. For example, why do we find the world the way it is? Why are the laws as they are? Why are the constants of Nature the way they are? Why are these ‘just so’? Why are we able to make sense of it and mathematize it? This line of questioning has traditionally pointed in two directions; directions that can often lead down intersecting paths. On the one hand one can use the comprehensibility and habitability of the Universe as a cosmological argument for the existence of a greater power who has made it so precisely for this reason. On the other hand one can simply point out that if it weren’t habitable and comprehensible then we wouldn’t be here to inhabit and comprehend it! We can unpack this latter suggestion in at least two ways:

- There is an ensemble of universes, each with different parameters and laws, not all of which are inhabitable and comprehensible. We occupy one of a small subset that is habitable and comprehensible.
- Again, a greater power must be responsible for our finding ourselves in such a nice situation—what Paul Davies has dubbed a “Goldilocks” situation.

In this paper I describe and defend an approach broadly similar to Tegmark’s mathematical universe hypothesis (see, e.g., [16]). His approach resembles old-fashioned Pythagoreanism, only with a multiverse component included that is absent in other Pythagorean accounts. There are several strands involved in getting what we want; namely an argument that has as its *explanandum* “there is existence” (or “something exists”). My account differs from Tegmark’s in that I am not so concerned with describing the way our Universe is, but rather the issue of why there is anything at all.<sup>2</sup>

### 3

### THE PHYSICS AND PHILOSOPHY OF MODALITY

At the root of my explanation of existence is the notion that it is not possible for there to be nothing: existence is necessary. In large part, our ordinary conceptions

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<sup>2</sup>The strategy I use, involving logic and a merger between physics and mathematics, bears some similarities too to Paul Benioff’s approach [4]. However, he defends the view that a theory of everything, and mathematical and physical reality ‘emerge together’ in some sense. I am steering clear of the notion of a theory of everything here. Here I think Gödel’s theorem would surely bite: for a theory will be a representation and any such mapping will be lossy. But that is not to say that any underlying reality of a logical theory is compromised; I’m simply referring to the distinction between the tools (i.e. theories) we use to represent reality and the reality itself. Gödel’s incompleteness theorem applies to the former alone. Indeed, this does impose a limitation on physics’ theoretical prowess in that if reality is a certain way (related to properties of arithmetic) then a complete account using any logico-mathematical representation will prove to be impossible. This is an epistemic limitation rather than a limitation imposed on reality.

of what is possible and what is impossible are demarcated by physics. It is the laws of physics that tell us what can and what can not be the case in our world. However, more generally, it is the job of laws of nature. Laws of nature specify the invariant quantities, and invariant quantities are those that cannot possibly be otherwise (that is, their denial would involve an inconsistency with the laws of physics). Note that this automatically transcends the bounds of the empirical, for laws in this strict sense represent a potentially infinite family of scenarios. Any talk of laws, in this strict sense, then, treads into deep metaphysical waters.

Laws of nature determine what is and is not possible in our world. However, we may wish to have a broader notion of possibility not so dependent upon the laws of present science. Philosophers call such physics-based accounts of possibility *nomological*. If we are concerned with what is possible from the point of view of logic itself, then we extend our domain to *metaphysical* or *logical* possibility. The only constraints here come from mathematical logic: if something is not inconsistent with basic logical laws (such as the law of non-contradiction) then it is possible; otherwise it is impossible (e.g. a square circle). In this paper we are clearly concerned with the stronger form of modality, with logical possibility and necessity. Philosophers ground their talk of possibility and necessity in possible worlds, the semantic representation of modal logic. Something is possible if there is at least one possible world in which it obtains. Impossibility is parsed as not obtaining in any possible worlds. Necessity is parsed as obtaining in all possible worlds. Mathematical truths are necessary truths: they hold at all possible worlds.

As we all know, in Shakespeare's most famous soliloquy, Hamlet considered the contingency of human existence: to be or not to be? There is the very real possibility of human non-existence (putting aside the fact that he is a fictional character, there is a possible world in which Hamlet is dead), and Hamlet mused over whether it mightn't be better to instantiate non-existence rather than existence. We are concerned with a generalized version of Hamlet's problem: does the contingency of human existence hold for the sum total of all existing things (where we place no restrictions such as 'physical' on the form of existence)? I argue that it does not, so long as we are willing to accept that reality is mathematical, following the most basic rules of logic. Before I get to this, I first describe and deflate an argument from the philosophical literature that is supposed to show that nothingness is a genuine possibility.

## 4

## AGAINST METAPHYSICAL NIHILISM

Metaphysical nihilism [MN] is the view that there could have been nothing at all; it views as possible the 'rather than nothing' component of the question 'why is there something rather than nothing?' Philosophers understand this possibility via an *empty* possible world (whether literal, abstract, or fictional); that is, one containing no objects or states of affairs whatsoever. We might wish to have such a world at

our disposal to act as a ‘truthmaker’ for the seemingly reasonable claim that our world, and indeed any world, might never have existed, that there might have been nothing. If we don’t have such a possible world at our disposal, then the existence of *something* is rendered a necessary truth. One can then use the difficulties in making sense of the possibility of nothingness in this sense to answer the question of why there is something rather than nothing. David Armstrong defends just such a position, writing that it isn’t “logically possible for there to be nothing at all” ([1], p. 24); as does David Lewis ([12], p. 73). The problem with this as an explanation of existence is that they are based on a particularly strong (and, to many, peculiar) account of possible worlds, known as a compositional account.

According to the compositional account of worlds [C], worlds are ‘composed’ out of some basic set of things: objects, states of affairs, etc. David Lewis [12], for example, defines a possible world as the biggest mereological sum of objects that are spatiotemporally related. David Armstrong [1] views possible worlds as recombinations of *actual* states of affairs. The received view seems to be that MN is incompatible with C. Armstrong is quite explicit about this, writing that the combinatorial approach “cannot countenance the empty world” since “the empty world is not a construction from our given elements” ([1], p. 63). I have no desire to defend C<sup>3</sup>, for I am not concerned here with the way the world is put together. However, the argument for MN (called the subtraction argument) provides a nice window into the explanation of existence that I do defend.

The ‘subtraction argument’ [S] is due to Tom Baldwin. S is thought to pose problems for C, since C implies that MN is false. MN is derived as follows (here condensing the original presentation in [3], p. 232):

1. There is a world with a *finite* number  $n$  of *concrete* objects (accessible from our own: i.e. possible relative to ours). Call this world  $w_n$ .
2. The existence of any object  $o$  in  $w_n$  is *contingent*.
3. The non-existence of  $o$  does not imply the existence of another object  $o'$ .
4. There is a world,  $w_{n-1}$ , accessible from  $w_n$  containing exactly one less object than  $w_n$ . There is a world accessible from  $w_{n-1}$ ,  $w_{(n-1)-1}$ , containing exactly one less object than  $w_{n-1}$ .
5. By iterating the above procedure (i.e. by repeated ‘subtractions’) we arrive at a world  $w_{n-m} = w_{min}$ , accessible from  $w_n$ , that contains exactly one object.
6. Therefore, by steps 2, 3, 4, from  $w_{min}$  there is an accessible world,  $w_{nil} = w_{n-m-1}$ , containing no objects at all (= MN).

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<sup>3</sup>In fact, on Lewis’ account, I think it is far too weak since it presupposes that the spacetime relation is fundamental. However, recent developments in physics appear to be converging on the view that this is not the case, and that spacetime is something that emerges at certain energy/length scales.

There are problems with this argument. Baldwin's restriction to *concrete* objects, so as to rule out spacetime points for example, is problematic and seems unfounded.<sup>4</sup> MN is an attractive view only if it refers to genuine nothingness, including the non-existence of spacetime points or regions (or the more fundamental structure underlying it). Indeed, it makes perfect sense to include these less concrete objects in the subtraction argument too, for it is possible to have universes of different (finite) sizes and, therefore, containing different numbers of points or regions.<sup>5</sup> Spacetime points though perhaps abstract, are contingent. However, there is a direct clash with the view that reality is mathematical for mathematical truths are necessary and therefore cannot ever be 'subtracted' to produce another possibility. If they didn't obtain a contradiction would result. Even if we could have an empty world in Baldwin's sense, or even in the stronger sense I alluded to earlier, it would not be empty of mathematical truths which, by their very nature, obtain in all possible situations.

## 5

## THE LOGIC OF EXISTENCE

This leads us, finally, to the explanation of existence. It can be expressed in a rather simple way, as one should expect given the immense generality (i.e. non-specific nature) of its explanandum. Indeed, we have already expressed the argument several times above. Either existence is contingent or it is necessary. If it is contingent then there is no complete coherent account of existence. If it is necessary then we need a necessary structure to ground this fact. Mathematical structures are of this kind. If reality is mathematical then it must exist. Reality is mathematical (as evidenced by the effectiveness of mathematics in the sciences). Therefore, there is existence. This is not a case of logic 'causing' existence, or bringing existence into being 'out of nothing'. Mathematical structures are timeless. We can add to this the fact that if this is the *only* reason that can be found for existence to be the case, then physical reality (or just reality) has to be mathematical. In other words, the universe is mathematical because there is existence, and the only reason for there to be existence is that there are mathematical truths.

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<sup>4</sup>Baldwin argues that concreteness is to be determined by the principle of the identity of indiscernibles: only concrete objects satisfy the principle. Unfortunately, this rules out the entire (particle) ontology of our own world, replete as it is with bosons and fermions, both of which fail to satisfy the principle—bosons violate a very strong form of the principle where even relational properties are included (see [15]; but see [14] for an argument that rescues PII from fermions).

<sup>5</sup>One might object that this will involve infinitely many objects and threaten the argument. However, there is a real possibility of discrete, quantized geometry at scales of the order  $10^{-34}$  cm (i.e. the Planck length). This would imply that a physical space with a finite radius is built up from a finite number of elementary volumes (see [2]).

## OBJECTIONS

6.1 *Neo-Megarianism*

The Megarian school<sup>6</sup> held that there were no such things as unactualized possibilities. No distinction was made between actuality and potentiality. The very definition of possibility, due to Diodorus Cronus, was written in terms that implied actuality at some time:

*p* is possible if and only if *p* either *is* the case or *p will be* the case

If the world is mathematical, then the basic Megarian definition would appear to be applicable (though the temporal element drops out). For the Megarian conception of possibility went hand in hand with their Eleatic principles: there is no change and there is no motion. All is one. Likewise, mathematical structures are atemporal and so fall under the same principles. From Boethius we have the Megarian views on necessity and related notions too:

Diodorus defines the possible as that which either is or will be ... the impossible as that which, being false, will not be true ... the necessary as that which, being true, will not be false .. and the non-necessary as that which either is already or will be false. [5], p. 234]

Paul Davies has described Tegmark's position as a total multiverse in which "anything goes" ([8], p. 14): anything possible is actual. This is pure Megarianism. However, this isn't quite right: there is the very strong constraint of mathematical consistency which any mathematical structure will automatically satisfy. At the root of this view is, then, consistency: one cannot have claims and their contraries as theorems or axioms.

6.2 *Arbitrariness*

In a similar way, one can dispose of another of Davies' objections to Tegmark's view, that it is somewhat arbitrary to classify elements of the multiverse on the basis of mathematical properties: "One might just as well postulate the set of all aesthetically appealing universes, the set of all virtual realities, the set of all emotional experiences, the set of all morally good actions, etc." (ibid., p. 15). But these other choices clearly do not have the strong basis in consistency. Nor do they have the atemporal element that is so critical. Moreover, the list of puzzles that are resolved by the view that reality is fundamentally mathematical is fairly impressive (far more impressive than if one replaced mathematical structures by aesthetic properties!). One can readily see why the universe is regular and why we can, therefore, make accurate predictions; why it is comprehensible (i.e. the unreasonable effectiveness of mathematics is made reasonable); and more importantly, why the universe exists at all.

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<sup>6</sup>See William and Martha Kneale's *The Development of Logic* ([11], p. 117) for information on the Megarians. See also Makin [13] for a direct examination of their notion of possibility.

### 6.3 Gödel's Theorem

There are other objections stemming from Gödel's incompleteness theorem that are somewhat harder to deal with—I sketched a response earlier in footnote 2. How can we base our explanation of existence on mathematical logic and mathematical structures when Gödel's incompleteness theorem tells us that within any axiomatic system there will be statements (or propositions) that cannot be deduced from the axioms and rules of inference. This is a deep result, and can be damaging to certain theses. However, it is no problem for the view presented above. The theorem does not tell us that there is any problem with mathematical truths *per se*; only that there is no algorithmic way of generating all such truths. The mathematical universe is safe from Gödel's theorem. We must distinguish between truth and provability. Alain Connes puts it this way:

What logic brings above all is a clear demonstration of the limitations of the formal axiomatic method, that is, logical deduction within a formal system. This intrinsic limitation leads us to separate what can be *proved* in a given logico-deductive system from what is *true* and that I will call, with deliberate provocation, “primordial mathematical reality.” [...] This primordial reality can be explored with conceptual tools whose limits logic enables us to better understand. [T]his tool that one constructs ... will never exhaust mathematical reality. The deductive tool and the primordial mathematical reality exist on two different levels, and it is an illusion to think that we can reconstruct this primordial mathematical reality on the basis of a deductive system. [[7], pp. 5–6]

And, of course, we don't need to for our purposes. Mathematical reality is a timeless and necessary reality. If physics manages to merge with this reality then we have an explanation of existence via physics. That we cannot have a complete representation of it is unfortunate, but beside the point. For Connes, our logico-mathematical representations are no more than lenses, offering a perspectival glimpse of the underlying reality.

### 6.4 Meaninglessness

Sean Carroll raises the following objection to *any* attempt to answer what Adolf Grünbaum called the ‘Primordial Existence Question’:

Ultimately, the problem is that the question “Why is there something rather than nothing?”—doesn't make any sense. What kind of answer could possibly count as satisfying? What could a claim like “The most natural universe is one that doesn't exist” possibly mean? As often happens, we are led astray by imagining that we can apply the kinds of language we use in talking about contingent pieces of the world around us to the universe as a whole. It makes sense to ask why this blog exists, rather than some other blog; but there is no external vantage point from which we can compare the relatively likelihood of different modes of existence for the universe. (<http://blogs.discovermagazine.com/cosmicvariance/2007/08/30/why-is-there-something-rather-than-nothing/>)

The problem of why there is something rather than nothing strikes many as a pseudo-problem. But it is, I think, a genuine problem: it is an undeniable fact that there is existence. How come? If it is a contingent matter that there is existence then that demands an explanation. If it is a necessary matter, then we have our explanation automatically given. Any contingency in explanation will



leave elements of incompleteness. Appealing to the necessity of existence is the only way to answer the problem satisfactorily. The kind of necessity we are after (namely that for which alternatives are inconceivable: logically impossible) is found in mathematics. Physics is heavily based on mathematics, and we face a problem understanding why that is, why mathematics seems to work so well in describing reality. If we make the hypothesis that reality is mathematical then that problem evaporates. But then physics is dealing with mathematical (necessary) truths. These are truths that have a necessary existence. The problem of why there is something rather than nothing does not apply to mathematical somethings: they simply couldn't be otherwise.

## 7

## FROM MATHEMATICAL PHYSICS TO PHYSICAL MATHEMATICS

The strategy I am advocating is that physics, in becoming more or less completely aligned to mathematics (in terms of content, at least)<sup>7</sup>, will be able to penetrate down the ladder of explanation to the very deepest rung of all: existence. We do not have the same kind of problem with the existence of mathematics. Mathematical statements are necessarily true in the sense that if they are true in one world (in the sense of modal logic) then they are true in all worlds. They are not created. They are not located in spacetime. The question of why is there something rather than nothing simply does not make sense if the somethings in question are mathematical.

The entanglement of mathematics with physics is hard to miss. As Dirac eloquently expressed it:

The steady progress of physics requires for its theoretical formulation a mathematics that gets continually more advanced. This is only natural and to be expected... Non-euclidean geometry and noncommutative algebra, which were at one time considered to be purely fictions of the mind and pastimes for logical thinkers, have now been found to be very necessary for the description of general facts of the physical world. It seems likely that this process of increasing abstraction will continue in the future and that advance in physics is to be associated with a continual modification and generalisation of the axioms at the base of the mathematics rather than with a logical development of any one mathematical scheme on a fixed foundation.

The theoretical worker in the future will therefore have to proceed in a more indirect way. The most powerful method of advance that can be suggested at present is to employ all the resources of pure mathematics in attempts to perfect and generalise the mathematical formalism that forms the existing basis of theoretical physics, and after each success in this direction, to try to interpret the new mathematical features in terms of physical entities. ([9], p. 66)

This simple quote gets to heart of the matter: no matter how crazy and inapplicable some piece of mathematics appears to be at first sight, nature seems to have an uncanny knack of making use of it. Indeed, what basic mathematical structures *can't* be seen as having *some* level of application in the natural world? The search

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<sup>7</sup>That is, they will in any case have different *uses* to which they are put, much as engineering can be viewed as a practical version of the knowledge from various 'pure' sciences.

for and exploration of mathematical structures collapses into the exploration of our Universe. The distinction between physical and abstract is dissolved.

Similarly, Chern expressed the kind of unified mathematical physics (or physical mathematics) that I have in mind here:

In 1975, impressed with the fact that gauge fields are connections on fiber bundles, I drove to the house of S. S. Chern in El Cerrito, near Berkeley ... I said I found it amazing that gauge theory are exactly connections on fiber bundles, which the mathematicians dreamt up without references to the physical world. I added "this is both thrilling and puzzling, since you mathematicians dreamed up these concepts out of nowhere." He immediately protested: "No, no. These concepts were not dreamed up. They were natural and real." [18], p. 567]

One can see a strong tendency for unification between mathematics and physics in recent years. Fields medals are routinely offered to physicists. The Clay Millennium prizes in mathematics involve many physics problems. If we take this as evidence that physics and mathematics have a common basis, then we get out some explanations of puzzling phenomena. Not only the comprehensibility of the universe, but also its very existence.

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