Flowing with a Frozen River
An Essay on the Nature of Time

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For discussing foundational questions, like the nature of time, we need a framework. Ideally, this would be provided by a Theory of Everything. Until the discovery of TOE, I propose a mathematical structure that can be used to represent theories of Physics in a unitary framework, similarly to the way in which the group actions represent various geometries in the Erlangen program. This construction extracts essential aspects of various theories, concerning the space, time, physical law, and causality. I introduce a causal structure and apply it to models of time and time travel. I propose an argument, based on causality, for the initial singularity of the Universe, and for the physical reality of gauge potentials (all three related in an unexpected way). Then, I discuss a new version of Quantum Mechanics, that replaces the discontinuous wavefunction collapse with delayed initial conditions, and has significant implications on time and causality. After presenting the arrows of time as emergent phenomena, I discuss the mind and its perception of time as flowing, in the context of the block spacetime. Then, I apply the previous observations to analyze the possibility of free-will. I propose a hypothesis about the free-will, and a crucial experiment that can confirm or reject it.

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The Frozen River and its flow

As our understanding of the Universe increases, the time becomes more and more geometric. Initially being only a parameter in the equations of Mathematical Physics, it proved to be more related to the space than we expected, mixing together in the structure named *spacetime*. With the Theory of Relativity, these geometric aspects started to suggest the image of a frozen time – the *block time* – named "frozen river" by Brian Greene.

On the other hand, we have the acute feeling that the time is flowing, that we are subjects experiencing this flow, and that we are free to choose our experiences. If the time is really frozen, then why do we perceive it as flowing? Is the frozen time idea compatible with the indeterminism? Is Physics compatible with the idea of free-will? But what is this free-will? I will present some thoughts about these questions, mainly based on my research [1–4].

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I. A MATHEMATICAL FRAMEWORK FOR FOUNDATIONAL QUESTIONS

A. Discussing foundations without a Theory of Everything

A foundational discussion should take place within a particular theory in Physics. Our most successful theories, the Standard Model of particle physics (SM) and the General Relativity (GR), seem to contradict one another. Several candidates try to fill this gap – String Theory, Quantum Gravity, Twistor Theory etc. – which are still under development, and propose opposite visions about time and space. It seems impossible to predict how the final theory will look like. Is spacetime a differentiable manifold, or a causal set, or a spin network, or a graph? Is time discrete, or continuous? This unknown makes the foundational discussions very hard. The following mathematical structure, which I present in more detail in [1], attempts to distill general principles and provide a unified framework for thinking about foundational questions.

B. Space, time, and physical law

We begin the distillation process with a differentiable manifold[1], representing the *spacetime*. In Newtonian and Quantum Mechanics, the spacetime is $\mathbb{R}^{3+1} = \mathbb{R}^3 \times \mathbb{R}$, space and time being fundamentally distinct. Special Relativity propose the four-dimensional Minkowsky space $\mathbb{R}^4_1$, also used in the Quantum Field Theory (QFT). General Relativity's spacetime is curved – a Lorentz manifold. The Kaluza-Klein and String theories is still a differentiable manifold, with more space dimensions.

Spacetime is populated with *matter fields* – scalar, vector, tensor, or spinor fields – all being particular vector fields. The physical *laws* are expressed by partial differential equations (PDE). A particular solution representing the matter can, in general, be identified by a set of initial/boundary conditions.

A first step is to think the spacetime as a differentiable manifold, and the matter fields as sections in differentiable vector bundles[2], satisfying appropriate PDE representing the physical laws. We can consider only one vector bundle, because if there are more, they can always, by direct sums and tensor products, be combined in a single vector space. The equations will also combine in a single equation.

This construction is natural in differential geometry. It represents well physical theories like wave mechanics, Electrodynamics, Quantum Mechanics, Newtonian and GR gravity, QFT and Gauge Theory. For topological or discrete theories, we need a generalization.

For the next step, we remember that the local sections of the vector bundle form a sheaf[3]. The local solutions of the PDE form a subsheaf of it. We allow the *spacetime* to be a more general topological space. We replace the PDE with a sheaf over the spacetime – the *law sheaf*. The *matter field* is a global section in the law sheaf.

One important property of the physical laws is that they are (topologically) identical in any point of the spacetime. In [1] I defined *locally homogeneous topological spaces*, which have the property of being identical in any two points, and *locally semi-homogeneous sheaves*, which are identical in any two identical points of the base space[4]. The manifolds and the sheaves of PDE are locally homogeneous. A reasonable condition for the law sheaf is the local homogeneity.

A *world* is a mathematical structure consisting in a locally homogeneous topological manifold (spacetime), and a locally homogeneous law sheaf. A world is *differentiable* if its spacetime and law sheaf are differentiable[5].

These constructions are reasonable for fields, but do they work in the case of particles? In Newtonian mechanics, a particle is point-like in space, and curve-like in spacetime. Such situations can be constructed by using the *Sierpińsky space* $\mathbb{S}$, which is a topological space formed by two points, $\{0, 1\}$, with the topology $\tau = \{\emptyset, \{1\}, \{0, 1\}\}$. Taking $\mathbb{S}$ as fiber, we can construct a bundle on the spacetime $\mathbb{R}^{3+1}$. Any continuous section f can be naturally identified with the characteristic function of the closed set $f^{-1}(0) \subseteq \mathbb{R}^{3+1}$. Each curve on the spacetime becomes a section of the Sierpińsky bundle. To represent Newtonian point-like particles, we keep in the law sheaf only curves respecting Newton's laws.

By taking an appropriate 10 or 26-dimensional space, strings instead of points, and the Nambu-Goto action or the Polyakov action as constrain for the law sheaf, we can construct the String Theory.

The discrete theories of space and time can be based on discrete topologies, graphs, simplicial complexes. Lattice gauge theory, Regge calculus, causal sets and spin networks are worlds of this type.

Since the topological properties of a World can be expressed in Category Theory, in [1] I proposed a generalization of the World structure, which is able to cover even more unorthodox theories in Physics.

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[1] A space that can be considered as being made of patches cut from $\mathbb{R}^n$ or $\mathbb{C}^n$, smoothly glued together.

[2] Vector bundles are manifolds obtained by smoothly gluing vector spaces at each point of other manifolds. Sections are generalizations of fields.

[3] A *sheaf* on a topological space X is a way of associating to each open set, a set of *sections*, allowing the sections to be restricted to smaller open sets, or glued together on larger sets.

[4] A locally semi-homogeneous sheaf on a locally homogeneous space is *locally homogeneous*.

[5] The differential structure on a topological manifold can be reconstructed from sheaves.

II. CAUSAL STRUCTURES

A. Causal and locally causal worlds

In [1], I defined a *causal world* as a world with a preorder compatible with the topological structure of the spacetime. The *time* is taken compatible with the causal relation. Two events may be: one in the causal past of the other, simultaneous, and causally independent. The *principle of causality* requires that any germs[6] in causally independent events can be extended to a global section.

In General Relativity the spacetime can contain causal loops. This suggests a generalization of the causal relation to a *local causal relation* – for which exists an open covering $\mathcal{U}$ of spacetime, such that its restriction to any open from $\mathcal{U}$ is a causal relation. The local causal relation is required to be recoverable from these restrictions.

A world is *deterministic* if any section is determined at any event by the germs in its past. If it can be determined by its germs at any given time instant in the past, it is a *strongly deterministic* world.

B. Branching and looping time. Time travel

In World Theory, the spacetime may split, as in the Many Worlds Interpretation of QM (hence, the spacetime is not Hausdorff separated). Any indeterministic world can be represented as a branching world, so they are mathematically equivalent. In particular, there is no difference between the indeterministic QM and the MWI.

The causal worlds are compatible with continuous and discrete times, linear time, branching time, and time loops. They are logically consistent, and fit perfectly in the *frozen time* paradigm.

The so-called time travel paradoxes contradict only our common sense. Causal worlds are compatible, in principle, with the possibility to change the past, if the matter field behaves similarly to holomorphic functions that become multivalued by analytical continuation. No paradox in creating alternative histories by *changing* the past.

But *affecting* the past implies causal loops – reasonable at microscopic level, but very improbable when entropy is significant enough. A microscopical causal loop is, for example, a creation, followed by annihilation, of a pair electron-positron – an electron looping in time. But more complex systems, like humans, looping in time, raise some problems. A time traveler looping in time, consequently in the phase space, is very improbable, being complex enough to be an irreversible system.

[6] The germs capture the local properties of a sheaf at a point.

III. A CAUSAL ARGUMENT

A. A strong causality principle

I propose the following speculative but interesting *strong causality principle* (SCP):

On the same spacelike hypersurface (corresponding to an instant of time t_0), the value of the matter field at a point must not depend on its value in other points.

It seems reasonable to forbid the necessity to choose the initial conditions globally – this seem to defy causality. Metaphorically, the Creator must ensure global constrains between the field's values at causally independent points. But many PDE of Physics violate this principle.

B. Strong causality and the initial singularity

Long time, the gauge fields were considered more fundamental, their potentials being viewed as without physical reality. The Aharanov-Bohm experiment proves the physical reality of the electromagnetic potential. I present here a different argument, based on SCP, for the Maxwell potential (works identically for Yang-Mills potentials).

Gauss' law saids that the electromagnetic field's values at an initial time t_0 depends on the values at other points. But this violates the strong causality.

A similar situation occurs in General Relativity. When reformulated by Arnowitt, Deser and Misner[5] as an initial value problem with the initial data on a spacelike hypersurface Σ, Einstein's equation splits in an evolution equation, and a constraint for the initial metric on Σ. This constraint violates SCP.

But if we allow the initial Σ to degenerate to a point (or even to a special type of surface), the initial conditions can be chosen independently, in accord to the Strong Causality Principle.

C. Strong causality and the gauge potentials

The second way out is to consider that the potential, and not the electromagnetic field, is fundamental. The values of the potential at different points in space, at the same time t_0, can be chosen freely. The electromagnetic field, obtained from the potential, will have all the properties required by Maxwell's equations.

For the General Relativity case, we require that the metric is obtained, via Einstein equation, from the stress-energy tensor of the gauge fields and currents, which in turn is obtained from the gauge potentials.

In conclusion, the Strong Causality Principle implies at least one of the two: an initial singularity for our Universe, and physical reality for Maxwell and Yang-Mills potentials.

IV. SMOOTH QUANTUM MECHANICS AND DELAYED INITIAL CONDITIONS

A. The discontinuous collapse in Quantum Mechanics

1. *The wavelike behavior of quantum systems*

A pure state of a quantum system is described by a complex wavefunction – a vector in the *state space*[7] $\mathcal{S}$, whose time evolution is described by the Schrödinger equation. States that are mixed, either because they are undetermined, or because are entangled with another system, are represented by a *density operator* on $\mathcal{S}$, evolving according to the Liouville - von Neumann equation.

2. *The quantum behavior of quantum systems*

To find a solution of the evolution equation, we need an initial condition, which can be determined by a measurement. We never measure the state, but an *observable*, which is represented by a selfadjoint operator on $\mathcal{S}$. The measurement always finds the observed system in an eigenstate of the observable.

3. *The discontinuous collapse problem*

A new measurement of the system, of a different observable, imposes a new eigencondition, which is not necessarily compatible with the first one. This is usually explained by admitting a discontinuous jump into the new eigenstate. This jump was never observed directly, we don't know when and how happens. We don't know why it doesn't violate the conservation laws, which are characteristic only to the evolution governed by a Hamiltonian.

B. The delayed initial conditions mechanism

1. *One-measurement case*

By performing only one measurement, we can determine the state, without needing the discontinuous collapse. When we measure the spin of an electron, we first choose a spatial direction $\nearrow$. The result is either $|\nearrow\rangle$ or $|\swarrow\rangle$. Supposing we obtain $|\nearrow\rangle$, this state was valid also before the measurement. Had we choose to measure the spin along a different direction $\uparrow$, the only possibilities would be $|\uparrow\rangle$ and $|\downarrow\rangle$, and the spin was never $|\nearrow\rangle$. When we choose the direction, we choose the past of the electron's spin.

This behavior remains unchanged even in the *delayed-choice experiment*, when we decide the measurement direction long time after the electron was emitted.

Any quantum system can take one measurement without requiring discontinuous collapse, just a choice of the initial conditions. The choice is made now, but applies for the initial conditions in the past.

2. *Two-measurements case*

The discontinuity is required between two measurements of the same system. The delayed-choice experiment works also in this case (figure 1).

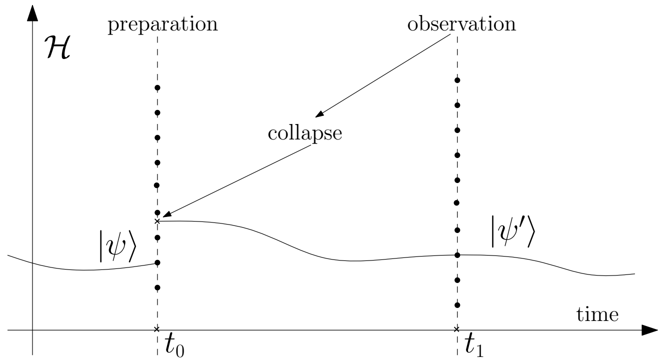

FIG. 1: The discontinuous collapse is required for two measurements. The initial condition seems to anticipate the outcome of the measurement.

When we measure the first time a system, the measurement device (which I will call *preparation device*) detects the state of the observed system, and then remains entangled with it. The second measurement is performed on this entangled system.

A measurement device is large, having macroscopic components, and its quantum state is undetermined. Because of the entanglement, now, the observed system is again in an undetermined state.

The second measurement doesn't really observe our system, but it's entanglement with the preparation device. This entangled state represents a superposition of many alternative possible interactions between the preparation device and the observed system. By choosing appropriate delayed initial conditions for the preparation device, can we "arrange" the things such that the second measurement device finds the system in an eigenstate. We apply the same mechanism of delayed initial conditions as in the one-measurement case, only this time is combined with the entanglement with the preparation device.

[7] The state space is a complex vector space with a Hermitian scalar product.

3. An example

A classical example, proposed to Bohr by Einstein, illustrates the mechanism. In the two-slit experiment, the wall with the two slits prepares the photons, and the screen detects their states as eigenstates of the position operator. According to Einstein, we can measure the momentum transferred to the two-slit wall by the photon, and detect which slit it used (figure 2).

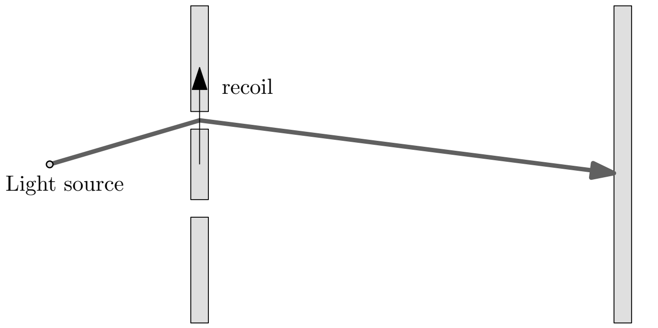

FIG. 2: If the the photon travels through the upper slit, the two-slits wall recoils upwards.

Bohr replied that this will destroy the interference, and both were right. In a delayed-choice version of this experiment, when we decide whether to observe the "which-way", or the "both-ways" behavior, we decide whether the wall changes its momentum with a definite value, or with an indefinite one (a complex superposition of momenta). We can decide after the photon has passed through the slits, what type of scattering it suffered, therefore we decide the state of the wall at that time. The wall must be in a state which guarantees the photon's evolution to the appropriate eigenstate.

Generally, we can measure twice a quantum system, as long as between them is disturbed by a larger device – the preparation device – in such a manner that it goes straight in the state obtained by the next measurement (figure 3).

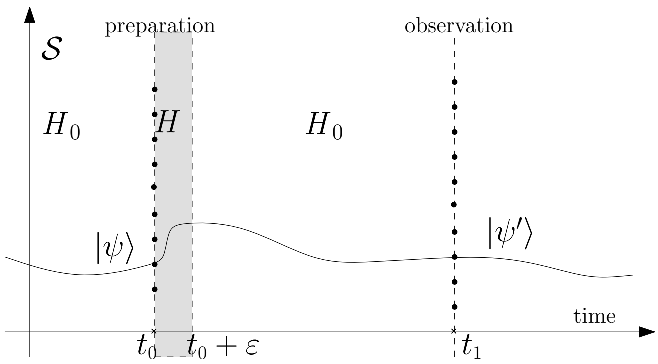

FIG. 3: The interaction with the preparation device changes the Hamiltonian to H between t_0 and $t_0+\varepsilon$, precisely to obtain the eigenstate $|\psi'\rangle$ as outcome of the second measurement.

4. The delayed initial conditions mechanism

In [2] I describe this mechanism in more technical detail. If the preparation leaves the system in a pure state, this one evolves according to Schrödinger's equation, updated with the interaction Hamiltonian. This Hamiltonian depends on the undetermined parameters of the preparation device. We have the following Quantum Control problem: choose the undetermined parameters of the interaction Hamiltonian such that the outcome of the next measurement is an eigenstate of the observable.

When the observed system gets entangled with the preparation device can loose its unitary evolution. The composite system, represented on the tensor product $\mathcal{S}_q \otimes \mathcal{S}_e$ between the state spaces of the observed system's and the preparation device, evolves governed by a unitary operator U. We can enlarge the state space $\mathcal{S}_e$ to ensure the purification of the states. The two measurements require that there is no entanglement at t_0 and t_1, therefore U must respect the condition

$$U(|\psi_0\rangle \otimes |\eta_0\rangle) = |\psi_1\rangle \otimes |\eta_1\rangle. \tag{1}$$

In general, a unitary operator like U puts the observed system in a mixed state, but in our case, the second measurement forces this state to be pure. For the mechanism to work, the equation (1) is required to admit solutions in $|\eta_0\rangle$ for each $|\psi_0\rangle$ and $|\psi_1\rangle$.

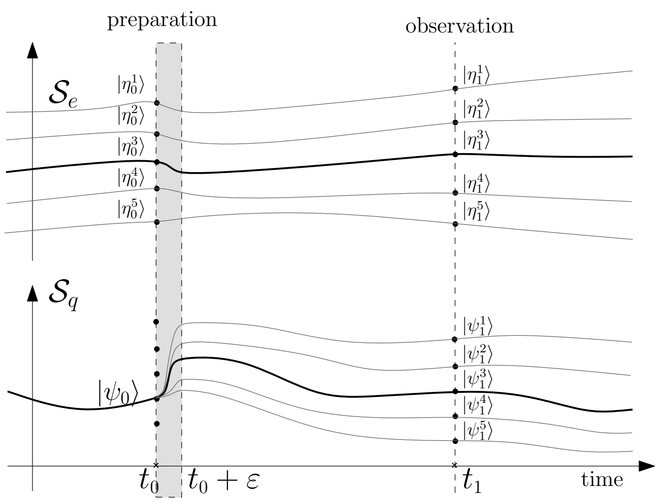

FIG. 4: Each possible outcome $|\psi_1^i\rangle$ can be obtained by choosing the appropriate states $|\eta_0^i\rangle$ representing the preparation device.

In the Smooth Quantum Mechanics, the wavefunction has physical reality, and the evolution is deterministic.

The particles are not point-like, but waves of different shapes, more or less localized in space. The quantum uncertainty resides only in the undetermined initial conditions.

V. THE FROZEN ARROWS OF TIME

A. Emergent phenomena

The physical laws can describe completely a computer, its states and processes, and consequently any software running on it. Yet, they fail to capture the inner logic of the software, which is independent of the material support. When running, the software is an additional constrain imposed to the computer's possible states. It is an *emergent phenomenon*: more elementary objects organized into something qualitatively different. The list also includes life, mind, morality, social behavior.

In World Theory, the emergent phenomena can be described as additional conditions on the law sheaf, similar to the initial conditions.

B. The arrows of time

The *thermodynamic time arrow* is an emergent law. It does not arise from the physical laws, rather from the highly improbable initial conditions of the Universe. The Universe evolves naturally to more probable states, and entropy increases.

The thermodynamic arrow is at the origin of the *psychological time arrow*: remembering the past is a thermodynamically irreversible process.

The *cosmological time arrow* – the Universe's expansion – also correlates with the thermodynamic arrow: the expansion from a small region, being divergent, is more probable than a convergent evolution.

Imagine how the steam on a window starts freezing from several centers, and then expand in all directions. One center is similar to the Big Bang. The freezing ripple represents the space at different instants of time, and it is more and more irregular – like entropy increases. We can deduce from the final, atemporal ice pattern, how it evolved in time. Similarly, the time arrows are compatible with the frozen time view.

C. A two-ways time paradox

Are possible, even in principle, islands of matter with reversed thermodynamics arrow? I propose, for amusement, the following thought experiment. Allen and Nella are from two planets with opposite time arrows. They have rooms in a spaceship, and share the kitchen. Allen enters the kitchen three times per day, finds food on the table, and eats. He wonders where the food come from, since the food supplies are finished weeks before. One day, he enters and finds Nella taking food with the spoon from her mouth, putting it on the plate, until she fills the plates just by "anti-eating". Allen realizes that each of them was eating what the other was anti-eating. They can survive without food supplies long time, but something seems not to add up.

VI. MINDS FLOWING IN THE FROZEN RIVER

A. Minds and algorithms

In [3] I showed that for any *algorithmic test* we can construct an algorithm that passes it. In particular, it is reasonable to ask the Turing test to be algorithmic, therefore it is, in principle, easy to pass, if we collect all the possible questions. I proposed a second test, which measures, in conjunction with the Turing test, our understanding of the human mind. Under inputs and outputs fixed by the Turing test, the shortest algorithm, having the best data compression, reveals the best understanding of the laws governing our minds.

B. A flow-zen time

We don't feel like software – we believe that, inside our body, there is some entity experiencing everything, feeling happiness, sadness, love, hope. Since all these feelings seem to be accompanied by brain processes, is our mind only Chemistry and Physics?

Imagine that we live in a time when we know all the physical laws, and we reduced all the brain processes to Physics, and all the mind processes to brain dynamics. Suppose that we are able to construct an algorithm simulating the mind in every detail. Nobody will be able to distinguish the algorithm simulating the mind of a person named, say, Leibcartes, from the original.

Any algorithm running on a computer can be described as a succession of states, and rules for changing the states as result of the inputs. Each state has data about other states, past or future. The thermodynamic arrow makes more reliable the data referring to past, than the data referring to the future.

The mind, like an algorithm, is a succession of states that are frozen at various instants in the block spacetime.

If our spacetime history is like a pickup disc, there is no pickup needle playing it, and marking the present. Each state of Leibcartes' mind perceives itself as being the real Leibcartes experiencing the time flow. Yet, all these states are nothing but slices in the spacetime block. We are a succession of frozen mind states, each of them naming its time "now", exactly like each human name its space position "here".

The time is flowing, and it is frozen.

C. Is mind deterministic or indeterministic?

If we feed with the same input data two copies of the computer simulation of Leibcartes' mind, will they behave identically? If the simulation algorithm is indeterministic, it can use the computer's random number generator to take random decisions. A deterministic algorithm will evolve identically on both computers.

VII. A FREE-WILL HYPOTHESIS

A. Indeterminism and free-will

Quantum Mechanics' acceptance is based partially on the introduction of a fundamental indeterminism in Physics. The fundamentally deterministic character of the previous theories was perceived as forbidding the free-will. Einstein did not accepted this feature, because "God doesn't play dice"[8]. Einstein, Schrödinger, de Broglie, Vigier, Bohm continued to hope for a deterministic QM, while Bohr, Heisenberg, Born accepted the indeterministim. The experimental rejection of Bell's inequality seemed to forbid any deterministic explanation, but Bohm's nonlocal theory of hidden variables.

In the Smooth Quantum Mechanics[2] the probabilistic character appears from the undetermined initial conditions, but the evolution is deterministic. The initial conditions play, if you want, the role of hidden variables, but without introducing extra objects to QM. Any choice allowed by the standard QM, like the choice of what observable to measure, is allowed also in the Smooth QM. Therefore, if the standard QM allows the free-will, so does the Smooth version. Both versions imply that the choice may affect the past, introducing delayed initial conditions, but the Smooth version pushes this past to the very beginnings. Therefore, in the indeterministic as well as in the Smooth QM, there is a *quantum freedom*.

If the quantum freedom plays a role in the brain processes, does this ensure the free-will? A similar question can be "does the computer's random number generator guarantee the free-will to an indeterministic algorithm?". Randomness has nothing to do with the free-will. If our minds are just algorithms, any decision is determined either by computations, or by the computer's random number generator. It seems that an algorithm cannot have its own free-will.

B. The last hope for the free-will

In a material world, everything happens either by deterministic evolution, or by randomness. It seems to be no place for the free-will. Following my paper[4], I will show that there is a last possibility, which can be in principle tested experimentally.

[8] I show in [1] that even Einstein's General Relativity can be indeterministic, since we can change locally the metric tensor.

Imagine a role-playing computer game – an algorithm, admitting input from human players. The player has some goals, and his success in reaching them is measured by a score. From the viewpoint of the algorithm, the human player, who feeds it with inputs, is nothing but a source of indeterminism, like the random number generator. But if the player inputs random commands, his score will be much bad than when he follows the goals.

The only hope for humans to have free-will is that the real "player", the real "I", is outside the physical world. This hypothesis cannot be easily accepted, having a serious problem: it introduces an "outside the world" explanation for our feeling of free-will, when this may be explained in future by a materialistic approach.

Yet, this problem is very important for us, as humans: we care if we are something more than an algorithm. If we reject the free-will, let's do it properly, by an experiment. The pot is too important to refuse to give it a chance.

Remember, we are in a future time, when Science evolved such that we can monitor all mind processes, at the physical level. Maybe at this time we already have a better proof for the free-will, but if we don't have, we return to our last hope: to prove that there is a "player" outside the physical world, able to select the initial conditions of this world, such that Leibcartes' mind takes the desired decisions at any time. Leibcartes' "I", the player, is atemporal, and exerts his free-will by acting directly to the initial conditions of the Universe, such that these conditions determine his physical manifestation to make the desired choices.

The test consists in checking the inputs of Leibcartes' mind: any interaction that can affect his decisions, any initial condition. If this input is random, and does not reveal an intention, then the free-will, and the "outside player", are fictions. If the input data "conspires" such that Leibcartes obtains high scores systematically (like the game player), then we can conclude that there is a player outside the physical world, which in fact makes the choices.

This hypothesis may sound too strange, but I think that it is the only way for the free-will to exist. It may look useless to postulate an influence from outside the spacetime, when the materialistic view seems to explain almost everything. Yes, but we can test it, and prove or disprove once for all the free-will hypothesis. Not now, then, when the Science will allow us.

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