# On the Non-Existence of TOEs


**Abstract**

Since the beginning of civilization, the ultimate goal of science has been to discover one theory that is capable of both describing and predicting all physical phenomena in the Universe. Over the centuries since that time, many theories have been presented as candidates for the Theory of Everything (TOE), but as yet not a single theory has been successful in describing all of the known laws of physics. In this paper, we outline a proof that TOEs cannot exist, and that at best we can search for approximations to the full laws of nature.


## 1 Introduction

The ultimate goal of physics, and in fact of all science, is to develop one theory that can fully describe and predict the Universe and everything it contains. Although there are a myriad of theories claiming to be Theories of Everything (TOEs), at the present time none are close to being proven correct - few if any would even satisfy the definition of 'Theory of Everything'.

For the purpose of this article, a TOE will be defined as a theory or model which:

(1) can be written down in a finite article,
(2) is capable of predicting either an exact state or a probability distribution for all possible states of the Universe and its contents without requiring any additional information, and
(3) is not consistent with any other state (or probability distribution) of the Universe. (ie. The theory predicts everything, with no further measurements needed to differentiate between different models)

It is worth noting that in this definition we do not require the TOE to describe the observed Universe, but only to be capable of describing every aspect of a possible Universe. This allows for the discussion of a set of all potential TOEs, without the limitation of being consistent with experimental results. Clearly though, the ultimate goal of science is to find a TOE which can describe our Universe. In this article it will be proven that there cannot exist any theory of everything, whether they describe our Universe or not.

## 2 The Non-Existence of TOEs

Consider a subset of the possible states of the Universe, in which each state consists of a spacetime of arbitrary dimension and containing N species of complex scalar fields[1] and K real constants (eg. particle masses and coupling constants). In this subset, the state of the Universe is determined[2] by the value of each field at each point in spacetime as well as the elements of the metric tensor.

Select one point in each spacetime. Since the subset under consideration must include all possible states of the Universe, it must also include states corresponding to every possible value of each scalar field and each constant at this chosen point. Since the elements of the subset must contain N values of a complex scalar field and K real parameters of the theory, the elements of this subset can be mapped onto the space $\mathbf{R}^K \times \mathbf{C}^N$ . As this space is known to be uncountable, the original subset is also uncountable, and therefore the set of all possible states of the Universe (denoted by $\mathbf{U}$) must also be an uncountable set.

Now consider the set of all possible TOEs, denoted by $\mathbf{T}$. By the condition that a viable TOE must be such that it can be written in a finite article, it must contain a finite number of characters. As a result, it is trivial to prove that the set of all TOEs is a countable set[3]. It also follows that the mapping

$$F : \mathbf{T} \to \mathbf{U}$$

[1] Although we assume scalar fields, this proof relies only on the degrees of freedom inherent in the fields and therefore can be generalized to fermionic fields and vector fields as well

[2] In this section we will present pure states in which all fields and constants are known. The case of superpositions of states or other theories involving probabilities will be reviewed in the next section.

[3] A simple proof is given in the appendix to this article

which maps each TOE to its predicted state of the Universe, cannot be a surjective mapping. As a result, there exist elements of **U** which cannot be described by any TOE. Furthermore, because the image of F is countable while **U** is not, the probability that a given state of the Universe cannot be described by any TOE is effectively 100%.

# 3 Quantum Mechanics & The Landscape Theory

The proof given in the previous section is presented in terms of a single well-defined state of the Universe. However quantum mechanics dictates that at best we can determine a probability distribution for the states[4], while the modern landscape theory suggests that the underlying theory itself and all relevant parameters are randomly selected.

In spite of the initial appearance of a failure of the proof, this strengthens the proof. Consider again a single point in a spacetime with a single scalar field. The probability distribution is a mapping from $\mathbf{C} \to [0, 1]$, such that each possible value of the scalar field is mapped to a real number between 0 and 1. Therefore a specific state of this single point micro-Universe consists of a surface in the space $\mathbf{C} \times [0, 1]$ instead of a single complex number. Since the set of all such surfaces is also uncountable, and since this is a subset of all possible probability distributions of states in the Universe, it follows that the set of all probability distributions of states of the Universe is also uncountable.

In landscape theories, the parameters of the theory are also randomly determined. Then each TOE predicts a surface in the space $\mathbf{R} \times [0, 1]$ for each parameter, corresponding to the probability distribution of selecting a specific value of the parameter in the landscape. Once again this is an uncountable set, and as before the set of all states of the Universe in a landscape theory is uncountable.

Therefore the proof given in the previous section remains valid when both quantum mechanics and landscape theories are included in the set of TOEs.

[4]For example, we cannot know both the position and the momentum of a particle to arbitrary precision, but instead must consider only the probabilities for certain values of these two quantities.

# 4 Conclusions

The proof presented in this article can have two interpretations. The first is that virtually all possible states of the Universe are indescribable by a TOE, and therefore it is effectively impossible that there exists a TOE which describes our Universe. Since the probability of a random Universe being described by a TOE is arbitrarily small, if such a TOE is eventually discovered then a new theory would be required to explain this fine-tuning of the Universe, which would then require a new theory and therefore the TOE would not actually describe 'everything'.

The second interpretation is that the possible states of the Universe must be further restricted to form a countable set[5]. In this case, the TOE violates our definition, in that restrictions must be added by hand and therefore the theory is not capable of predicting every aspect of the Universe.

The final result of this proof is that there cannot exist a theory of everything which is capable of describing every aspect of the Universe, and thus science will never be complete.


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# A Counting TOEs

In order to prove that the set of all TOEs is countable, we will first provide an algorithm for indexing all theories.

1. Assign every character a number (such as the ASCII codes for letters and numbers, a second copy for greek letters, a third copy for

[5] For example, if we assume spacetime has a discrete structure, that the values of the fields are themselves quantized, and that all masses and coupling constants are restricted to discrete values.

mathematical symbols, etc.)

2. Take the first letter of the article describing the theory, convert it to a number, $n_1$, using the coding above

3. Take the second letter, convert it to number $n_2$, and continue for all letters and symbols in the article

4. The index of this theory is defined as $I = 2^{n_1}3^{n_2}5^{n_3}...p_i^{n_i}$ where $p_i$ is the $i^{th}$ prime number.

This index is unique to each article (although only a very small number of indices actually correspond to valid theories). Since the TOEs can be 1-1 mapped to the integers, the set of all TOEs is countable.