

A Case Study of the Geometrical Nature of Exceptional Theories of Everything

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Abstract

In 2007, A. Garrett Lisi published a paper on "An Exceptionally Simple Theory of Everything". In response, this paper will explore the application of even-ranked Exceptional Lie algebras to the search for a Theory of Everything (TOE). Particular attention is given to the classic E_8 , and quasi-Exceptional, semi-simple E_{10} , E_{12} , and $E_8 \times H_4$ algebras. The complete skeletal framework of a TOE including fermion, boson and scalar boson content is presented for these algebras.

1. Introduction – Why Even-Ranked “Exceptional” Groups Are Special

Prior to A. Garrett Lisi’s [1] groundbreaking paper on the E_8 Exceptionally Simple Theory of Everything (TOE), most of the approaches towards Grand Unified Theories (GUT’s) or TOE’s involved either Special Unitary Lie Algebra Groups $SU(n+1)$ or Special Orthogonal Lie Algebra Groups $SO(2n)$ and $SO(2n-1)$, where $n = \text{Rank}$.

The Special Unitary groups were good for Yang-Mills “Boson GUT’s” that described interactions of the fundamental forces; such as the Standard Model $SU(3)_C \times U(1)_Y \times SU(2)_L$ of order eight plus one plus three, the Georgi-Glashow $SU(5)$ GUT of order 24 [2], and the Georgi $SU(11)$ GUT of order 120 [3].

And the Special Orthogonal groups seemed appropriate for describing “Fermion GUT’s” that define fermion particle multiplets such as Georgi’s $SO(10)$ with an order of 45 (technically an $SO(10) \times SO(10)$ of order 90 to include spin), and able to contain three generations of Standard Model fermions such as $(u_{L,R}^{r,g,b}, e_{L,R}, d_{L,R}^{r,g,b}, \nu_{eL})$, $(c_{L,R}^{r,g,b}, \mu_{L,R}, s_{L,R}^{r,g,b}, \nu_{\mu L})$, and $(t_{L,R}^{r,g,b}, \tau_{L,R}, b_{L,R}^{r,g,b}, \nu_{\tau L})$; although the right-handed neutrinos can also be included as singlet states. Previously, the author [4] noticed an interesting near pattern in certain Exceptional Lie algebras, as stated in Postulate 1 and shown in Table 1.

Postulate 1 – This pattern in even-ranked “Exceptional” Lie Algebra orders is given by Order = $N = n \times [2 \times SO(n/2 + 2) + 1] = (n^3 + 6n^2 + 12n)/4$, with a rank of n (except for E_6).

The actual order for E_6 is 78, which implies a less dense packing than this E_6' . This may have an analogy with the icosahedral group in which the icosahedron with 20 lattice sites has the same symmetry group as the dodecahedron with 12 lattice sites. Note the similarities between this “Composition” pattern in Table 1 and the progression of $SO(n)$ algebras: 3, 6, 10, 15, 21, 28, etc. The Coxeter-Dynkin diagrams of these new quasi-Exceptional groups are given in Figure 1. The 672 (1,008) roots of E_{12} (E_{14}) resemble a product of two (three) of Klein’s $\chi(7)$ modular curves with 336 triangles each [5] or 10-dimensional laminated lattices Λ_{10} with the same order [6]. The special feature of these even-ranked classic Exceptional groups such as G_2, F_4 or E_8 and quasi-Exceptional groups such as $E_6', E_{10}, E_{12}, E(2n)$, etc. is that these Exceptional groups of rank- n are large enough to contain both a Special Unitary and a Special Orthogonal group of rank- n . This makes these Exceptional groups perfect candidates for a Theory of Everything

Table 1 – A Pattern in Certain Quasi-Exceptional Lie Algebras

n	Name	$N = \text{Order}$	Composition
2	G_2	14	$2 \times (2 \times 3 + 1)$
4	F_4	52	$4 \times (2 \times 6 + 1)$
6	E_6 vs. $E_6'^i$	78 vs. 126	$6 \times (2 \times 6 + 1)$ vs. $6 \times (2 \times 10 + 1)$
8	E_8	248	$8 \times (2 \times 15 + 1)$
10	E_{10}	430	$10 \times (2 \times 21 + 1)$
12	E_{12}^{ii} [7]	684	$12 \times (2 \times 28 + 1)$
14	E_{14}	1,022	$14 \times (2 \times 36 + 1)$

Figure 1 – Coxeter-Dynkin Diagrams for Select Quasi-Exceptional Groups

Name	Extended Dynkin	Coxeter-Dynkin
$E_6' [5, 3^{2,1,1}]$	$E_{6R}' = 6 \times (SO(5) + 2 \times 4 + 2) = 120$ $1 \times 2 - 3 - 4 - 4 - 2$ $\quad \quad \quad $ $\quad \quad \quad 4$	
$E_{10} [5, 3^{5,2,1}]$	$E_{10R} = 10 \times (SO(9) + 4 + 1 + 1) = 420$ $1 - 2 - 3 - 4 - 5 - 6 - 7 - 8 - 4$ $\quad \quad \quad \quad \quad \quad \times$ $\quad \quad \quad 1 \quad \quad \quad \uparrow$	
$E_{12} [5, 3^{5,2,2,1}]$	$E_{12R} = 12 \times (SO(9) + 8 + 6 + 4 + 2 \times 1) = 672$ $1 - 2 - 3 - 4 - 5 - 6 - 7 - 8 - 8 - 6 - 4$ $\quad \quad \quad \quad \quad \quad \times$ $\quad \quad \quad 1 \quad \quad \quad \uparrow$	
$H_4 [5, 3^2]$ $\times E_8 [3^{4,2,1}]$	$H_{4R} = 4 \times (10 + 4 \times 5) = 120$ $5 - 5 - 10 - 5$ $\quad \quad \quad \times$ $\quad \quad \quad 5$	

ⁱ This E_6' may represent the union of an E_6 containing 72 roots with its dual E_6^* containing 54 roots.

ⁱⁱ It has been suggested that this E_{12} should be called K_{12}' to tie in with the Coxeter-Todd K_{12} lattice Ref [6,7] with 756 roots and similar symmetries, but the author previously called this E_{12} in Ref's [4,5].

(TOE) that contains a Special Unitary "GUT #1" and a Special Orthogonal "GUT #2" (see Postulate 2, Equation 1 and Table 2).

Postulate 2 – These even-ranked Exceptional Lie Algebras may decay into a product of Special Unitary and Special Orthogonal Algebras of equal rank times an Exceptional Algebra of smaller rank.

$$\begin{aligned}
 E(n)_{R+B} &\rightarrow SU(n+1) \times SO(2n) \times E(n-4)_R \text{ or} \\
 E(n)_{R+B} &\rightarrow \underbrace{A(n)}_{\text{TOE}} \times \underbrace{D(n)}_{\text{GUT\#1}} \times \underbrace{E(n-4)}_{\text{GUT\#2}} \times \underbrace{R}_{\text{Multiplet}}
 \end{aligned}
 \tag{1}$$

Table 2 – Exceptional Groups and Sub-Groups

n	Name	$E(n)_{R+B}$	$SU(n+1)$	$SO(2n)$	Difference
2	G_2	$14 = 2 \times 7$	$8 = 2 \times 4$	$6 = 2 \times 3$	0
4	F_4	$52 = 4 \times 13$	$24 = 4 \times 6$	$28 = 4 \times 7$	0
6	E_6'	$126 = 6 \times 21$	$48 = 6 \times 8$	$66 = 6 \times 11$	$G_{2R} = 12 = 1 \times 12$
8	E_8	$248 = 8 \times 31$	$80 = 8 \times 10$	$120 = 8 \times 15$	$F_{4R} = 48 = 3 \times 16$
10	E_{10}	$430 = 10 \times 43$	$120 = 10 \times 12$	$190 = 10 \times 19$	$E_{6R}' = 120 = 6 \times 20$
12	E_{12}	$684 = 12 \times 57$	$168 = 12 \times 14$	$276 = 12 \times 23$	$E_{8R} = 240 = 10 \times 24$
14	E_{14}	$1022 = 14 \times 73$	$224 = 14 \times 16$	$378 = 14 \times 27$	$E_{10R} = 420 = 15 \times 28$

Here, n is the rank of the even-ranked Exceptional TOE, Special Unitary and Special Orthogonal groups; the subscript R is the number of roots $n^3/4 + 3n^2/2 + 2n$ in a quasi-Exceptional group $E(n)_R$; and the subscript $R+B$ is the number of roots plus basis $n^3/4 + 3n^2/2 + 3n$ in a quasi-Exceptional group $E(n)_{R+B}$. Note the emergence of a new "Difference" term $E(n-4)_R$ in Table 2 that may be interpreted as a fundamental fermion multiplet for $n \geq 6$. Combining ideas from Postulates One and Two and Figure 1, we realize that these new quasi-Exceptional algebras are previously unidentified semi-simple Lie algebras:

$$\begin{aligned}
 E4[3^{0,2,1}] \times H2[5] &\rightarrow E6'[5,3^{2,1,1}] \rightarrow A6[3^5] \times D6[3^{3,1,1}] \times G2[5], \\
 E8[3^{4,2,1}] \times H2[5] &\rightarrow E10[5,3^{5,2,1}] \rightarrow A10[3^9] \times D10[3^{7,1,1}] \times E6'[5,3^{2,1,1}], \\
 E8[3^{4,2,1}] \times H4[5,3^2] &\rightarrow E12[5,3^{5,2,2,1}] \rightarrow A12[3^{11}] \times D12[3^{9,1,1}] \times E8[3^{4,2,1}], \text{ and} \\
 E8[3^{4,2,1}] \times H4[5,3^2] \times I2[7] &\rightarrow E14[7,5,3^{5,2,2,1,1}] \rightarrow A14[3^{13}] \times D14[3^{11,1,1}] \times E10[5,3^{5,2,1}].
 \end{aligned} \tag{2}$$

2. $SU(N+1)$ Boson GUT's

The Yang-Mills Boson GUT's represented by the Special Unitary groups are assumed to closely follow the author's prior efforts at a Boson GUT [4] with certain groups being very important, such as a 2-dimensional $SU(3)_C$ of Color and a 4-dimensional Georgi-Glashow $SU(5)_{GUT} \rightarrow SU(3)_C \times U(1)_Y \times SU(2)_L + SO(4)_X + SO(4)_Y$ that includes Electroweak.

Postulate 3 – There is an evident similarity between certain crystal symmetries and certain $SU(N)$ Lie algebra symmetries.

Postulate 3.1 – There is an evident similarity between the Tetrahedral Conjugacy classes and the Georgi-Glashow $SU(5)$ Boson GUT Lie algebra symmetries (see Equation 3).

$$\begin{pmatrix} x & C_3 & C_3 & S_4 & \sigma_d \\ C_3 & C_3 & C_3 & S_4 & \sigma_d \\ C_3 & C_3 & C_3 & S_4 & \sigma_d \\ S_4 & S_4 & S_4 & I & C_2 \\ \sigma_d & \sigma_d & \sigma_d & C_2 & C_2 \end{pmatrix} \Rightarrow \begin{pmatrix} x & g & g & X & Y \\ g & g & g & X & Y \\ g & g & g & X & Y \\ X & X & X & B & W \\ Y & Y & Y & W & W \end{pmatrix} \Rightarrow \begin{pmatrix} x & g & g & X & Y \\ g & g & g & X & Y \\ g & g & g & X & Y \\ X & X & X & \gamma^0 & W^- \\ Y & Y & Y & W^+ & Z^0 \end{pmatrix} \tag{3}$$

Equation 3 Legend – C_2 = Rotation by 180° (degree 2), C_3 = Rotation by 120° (degree 3), I = Identity, S_4 = Rotoreflexion by 90° (degree 4), σ_d = reflection in a plane through two rotation axes (degree 4), B = Weak Hypercharge of $U(1)_Y$ (degree 1), g = Gluons of $SU(3)_C$ (degree 3), γ = Photon, W = Neutral and Charged W 's of $SU(2)_L$ (degree 2), $X^{\frac{2}{3},c\bar{w}}$ & $Y^{-\frac{1}{3},c\bar{w}}$ = Leptoquark bosons (degree 4) of a Georgi-Glashow $SU(5)$ GUT, and Z = Neutral Weak IVB.

Here, the Strong force is placed in the top left position, followed along the diagonal by the next stronger Electromagnetic force (related to Weak Hypercharge), followed by the weaker Weak

force in the bottom right position. The Georgi-Glashow $SU(5)$ GUT has an order of 24 as does the total number of tetrahedral conjugacy classes. The Strong force has degree and order of three and eight as does C_3 , the class of tetrahedral rotations by 120° . All other sub-symmetries follow similar comparisons. These reflection symmetries S_4 and σ_d have a higher degree of symmetry than the rotation symmetries C_2 and C_3 , and are, therefore, intentionally placed in off-diagonal positions, thus representing higher rank terms. The B and W names reflect the unbroken Electroweak symmetries. After Spontaneous Symmetry Breaking of the Electroweak symmetry, these (B^0, W^0, W^\pm) mix quantum states to become (γ^0, Z^0, W^\pm) .

Another important Boson GUT is a 6-dimensional $SU(7)_{GUT} \rightarrow SU(5)_{GUT} \times U(1)_\phi \times SU(2,2)_{HF} + 8V$ (based on Octahedral symmetries) that introduces Hyperflavor (more details are presented in Chapter 5). These new algebras decompose via $U(1)_\phi \times SO(2,4)_{HF} \rightarrow U(1)_\phi \times SU(2)_C \times SO(4)_D \times SO(4)_E \rightarrow 2H + 2z + 4w + 2w^0 + 4w' + 2w'^0$.

Postulate 3.2 – There is an evident similarity between the Octahedral Conjugacy classes and a proposed $SU(7)$ Boson GUT Lie algebra symmetries (see Equation 4).

$$\begin{pmatrix} x & C_3 & C_3 & S_4 & \sigma_4 & S_6 & S_6 \\ C_3 & C_3 & C_3 & S_4 & \sigma_4 & S_6 & S_6 \\ C_3 & C_3 & C_3 & S_4 & \sigma_4 & C_2 & C_4 \\ S_4 & S_4 & S_4 & I & C_4^2 & C_2 & C_4 \\ \sigma_4 & \sigma_4 & \sigma_4 & C_4^2 & C_4^2 & C_2 & C_4 \\ S_6 & S_6 & C_2 & C_2 & C_2 & i & \sigma_h \\ S_6 & S_6 & C_4 & C_4 & C_4 & \sigma_h & \sigma_h \end{pmatrix} \Rightarrow \begin{pmatrix} x & g & g & X & Y & V & V \\ g & g & g & X & Y & V & V \\ g & g & g & X & Y & D & E \\ X & X & X & B & W & D & E \\ Y & Y & Y & W & W & D & E \\ V & V & D & D & D & \phi & C \\ V & V & E & E & E & C & C \end{pmatrix} \Rightarrow \begin{pmatrix} x & g & g & X & Y & V & V \\ g & g & g & X & Y & V & V \\ g & g & g & X & Y & w & w' \\ X & X & X & \gamma & W & w & w' \\ Y & Y & Y & W & Z & w^0 & w'^0 \\ V & V & w & w & w^0 & z & H_H \\ V & V & w' & w' & w'^0 & H_p & z \end{pmatrix} \quad (4)$$

Equation 4 Legend – Same as Equation 3 plus: C_4^2 = Rotation by 180° about a 4-fold axis, C_4 = Rotation by 90° (degree 4), i = Inversion, S_6 = Rotoreflexion by 60° (degree 6), σ_h = reflection in a plane perpendicular to a 2-fold axis (degree 2), σ_4 = reflection in a plane perpendicular to a 4-fold axis (degree 4), C, D, E, w and z = Hyperflavor bosons (degree 4), H, ϕ = Higgs, and $V^{\pm 1/2} = SU(7)$ Grand “bosons” (degree 6).

Other important Yang-Mills Boson GUT’s include an 8-dimensional pseudo- $SU(9)_{GUT} \rightarrow SU(7)_{GUT} \times U(1)_G \times$ pseudo $SO(3)_{WG} + 12T + 16$ (or 18) U that introduces Gravity, a simplified WIMP-Gravity, and a near unified CKM-PMNS Matrix (the top left 6×6 of Equation 15 in Section 5.2) and a 10-dimensional $SU(11)_{GUT} \rightarrow SU(7)_{GUT} \times U(1)_G \times SO(5,1)_{WG} + 12R + 12S + 12T + 20U$ (based on Icosahedral symmetries) that completes WIMP-Gravity (see Section 6.3) and the Unified CKM-PMNS-Munroe Mixing Matrix (see Section 5.2). Howard Georgi [3] has also written about an $SU(11)$ Boson GUT.

Postulate 3.3 – There is an evident similarity between the Icosahedral Conjugacy classes and a proposed $SU(11)$ Boson GUT Lie algebra symmetries (see Equation 5).

$$\left(\begin{array}{cccccccccccc} x & C_3 & C_3 & C_3 & C_3 & C_5 & C_5 & S_{10}^3 & S_{10}^3 & S_{10} & S_{10} \\ C_3 & C_3 & C_3 & C_3 & C_3 & C_5 & C_5 & S_{10}^3 & S_{10}^3 & S_{10} & S_{10} \\ C_3 & C_3 & C_3 & C_3 & C_3 & C_2 & C_2 & S_{10}^3 & S_{10}^3 & S_{10} & S_{10} \\ C_3 & C_3 & C_3 & I & C_5 & C_2 & C_2 & S_6 & S_6 & C_5^2 & C_5^2 \\ C_3 & C_3 & C_3 & C_5 & C_5 & C_2 & C_2 & S_6 & S_6 & C_5^2 & C_5^2 \\ C_5 & C_5 & C_2 & C_2 & C_2 & C_5 & C_2 & S_6 & S_6 & C_5^2 & C_5^2 \\ C_5 & C_5 & C_2 & C_2 & C_2 & C_2 & C_2 & S_6 & S_6 & S_6 & S_6 \\ S_{10}^3 & S_{10}^3 & S_{10}^3 & S_6 & S_6 & S_6 & S_6 & i & \sigma & \sigma & \sigma \\ S_{10}^3 & S_{10}^3 & S_{10}^3 & S_6 & S_6 & S_6 & S_6 & \sigma & \sigma & \sigma & \sigma \\ S_{10} & S_{10} & S_{10} & C_5^2 & C_5^2 & C_5^2 & S_6 & \sigma & \sigma & \sigma & \sigma \\ S_{10} & S_{10} & S_{10} & C_5^2 & C_5^2 & C_5^2 & S_6 & \sigma & \sigma & \sigma & \sigma \end{array} \right) \Rightarrow \left(\begin{array}{cccccccccccc} x & a & a & a & a & c & c & T & T & R & R \\ a & a & a & a & a & c & c & T & T & R & R \\ a & a & a & a & a & d & d & T & T & R & R \\ a & a & a & b & c & d & d & U & U & S & S \\ a & a & a & c & c & d & d & U & U & S & S \\ c & c & d & d & d & c & d & U & U & S & S \\ c & c & d & d & d & d & d & U & U & U & U \\ T & T & T & U & U & U & U & G & F & F & F \\ T & T & T & U & U & U & U & F & F & F & F \\ R & R & R & S & S & S & U & F & F & F & F \\ R & R & R & S & S & S & U & F & F & F & F \end{array} \right) \quad (5)$$

Equation 5 Legend – Same as Equations 3 and 4 plus: C_5 = Rotation by 72° about a 5-fold axis (degree 5), C_5^2 = Rotation by 144° about a 5-fold axis (degree 5), S_{10} = Rotoreflexion by 36° (degree 10), S_{10}^3 = Rotoreflexion by 108° (degree 10), σ = reflection (degree 2), a = “Color” 20-plet, $b = U(1)_Y$ “Photon”, c = “Higgs-Weak” 12-plet, d = Hyperflavor $SO(2,4)$, F = “Fifthons” = WIMP-Gravitons, G = Graviton, R, S, T and $U = SU(11)$ Grand bosons.

More speculative Boson GUT’s include a 12-dimensional $SU(13)_{GUT} \rightarrow SU(11)_{GUT} \times SU(3)_{3Gen} + SO(5)_N + SO(5)_P$ (see Table 3) that introduces new dimensions apparently responsible for three Fermion generations, and a 14-dimensional $SU(15)_{GUT} \rightarrow SU(11)_{GUT} \times SU(5)_{5Gen} + SO(5)_N + SO(5)_P + SO(5)_{N'} + SO(5)_{P'}$ that introduces new dimensions that imply the possibility of five Fermion generations.

Table 3 – An $SU(13)$ Boson GUT

12-D $SU(13)$												
10-D $SU(11)$												
8-D $SU(9)$												
6-D $SU(7)$												
4-D $SU(5)$												
I	$g^{g\bar{r}}$	$g^{b\bar{r}}$	$X^{\bar{r}-}$	$Y^{\bar{r}-}$	$V_L^{\wedge-}$	$V_L^{\vee-}$	T_1^{0*}	T_2^{0*}	R_1^-	R_4^+	N_1^*	N_6^*
$g^{r\bar{g}}$	g^3	$g^{b\bar{g}}$	$X^{\bar{g}-}$	$Y^{\bar{g}-}$	$V_R^{\wedge-}$	$V_R^{\vee-}$	T_4^0	T_3^{0*}	R_2^-	R_5^+	N_2^*	N_7^*
$g^{r\bar{b}}$	$g^{g\bar{b}}$	g^8	$X^{\bar{b}-}$	$Y^{\bar{b}-}$	w^{0*}	w'^{0*}	T_5^0	T_6^0	R_3^-	R_6^+	N_3^*	N_8^*
X^{r+}	X^{g+}	X^{b+}	γ	W^+	w_1^+	$w_1'^+$	U_1^{c-}	U_4^{c-}	S_1^{c+}	$S_4^{\bar{c}-}$	N_4^*	N_9^*
Y^{r+}	Y^{g+}	Y^{b+}	W^-	Z^0	w_2^+	$w_2'^+$	U_2^{c-}	U_5^{c-}	S_2^{c+}	$S_5^{\bar{c}-}$	N_5^*	N_{10}^*
$V_L^{\wedge+}$	$V_R^{\wedge+}$	w^0	w_1^-	w_2^-	z_1^0	H_H	U_3^{c-}	U_6^{c-}	S_3^{c+}	$S_6^{\bar{c}-}$	P_1^*	P_6^*
$V_L^{\vee+}$	$V_R^{\vee+}$	w'^0	$w_1'^-$	$w_2'^-$	H_P	z_2^0	$U_7^{\bar{c}+}$	$U_8^{\bar{c}+}$	$U_9^{\bar{c}+}$	U_{10}^{c+}	P_2^*	P_7^*
T_1^0	T_4^{0*}	T_5^{0*}	$U_1^{\bar{c}+}$	$U_2^{\bar{c}+}$	$U_3^{\bar{c}+}$	U_7^{c-}	G	F_1	F_A^*	F_D^*	P_3^*	P_8^*
T_2^0	T_3^0	T_6^{0*}	$U_4^{\bar{c}+}$	$U_5^{\bar{c}+}$	$U_6^{\bar{c}+}$	U_8^{c-}	F_2	F_3	F_C^*	F_B^*	P_4^*	P_9^*
R_1^+	R_2^+	R_3^+	$S_1^{\bar{c}-}$	$S_2^{\bar{c}-}$	$S_3^{\bar{c}-}$	U_9^{c-}	F_A	F_C	F_8	F_E^*	P_5^*	P_{10}^*
R_4^-	R_5^-	R_6^-	S_4^{c+}	S_5^{c+}	S_6^{c+}	$U_{10}^{\bar{c}-}$	F_D	F_B	F_E	F_{15}	Q_1	Q_4
N_1	N_2	N_3	N_4	N_5	P_1	P_2	P_3	P_4	P_5	Q_2	Q_3	Q_6
N_6	N_7	N_8	N_9	N_{10}	P_6	P_7	P_8	P_9	P_{10}	Q_5	Q_7	Q_8

Table 3 Legend: F = Fifthons = WIMP-Gravitons; g = Gluons; G = Graviton; γ = Photon; I = Identity; N , P and Q = Generation Bosons; R , S , T and U = $SU(11)$ Grand Bosons; V = $SU(7)$ bosons, W_1 and Z = Weak IVB's; Other w 's and z 's = Hyperflavor Weak IVB's; X and Y = Leptoquark Bosons; and c = color (r, g, b).

The $SO(2n)$ groups are the set of operators that interconnect a $2n$ -plet of fundamental particles. Combining this interpretation with the anticipated Fermion multiplets $E(n-4)_R$ yields the multiplet and generational structures listed in the “Difference” column of Table 2. Clearly, the first serious candidate for a TOE is $E8$ with three generations, each containing 16 particles.

In the author’s opinion, all Exceptional groups with $E(n)$ $n \geq 14$ should be ruled out as possible TOE’s because of the seven-fold symmetries of the $E(n-4)_R$ multiplet – this idea will be developed more fully later. Consequently, the most interesting even-ranked Exceptional TOE’s are $E8$, $E10$, $E12$ and $E8 \times H4$. Some of the literature refers to $E10$ and $E12$ as infinite dimensional Kac-Moody algebras. This paper will refer to the quasi-exceptional $E10$ and $E12$ algebras with rank and order defined in Tables 1 and 2. Next, we will analyze the properties of $SO(2N)$ Lie algebras.

3. $SO(2N)$ Boson GUT’s

Postulate 4 – Our $SO(2N)$ Boson GUT’s are assumed to be partially composed of an $SU(N+1)$ Boson GUT and a scalar multiplet consisting of Higgs/ Goldstone-like particles and longitudinal degrees of freedom (dof’s).

$$\begin{aligned}
 SO(8) &\rightarrow SU(5)_{GUT} + (H_L, \Phi_Z, \Phi_{W_1}, \Phi_{W_1}^*) \\
 SO(12) &\rightarrow SU(7)_{GUT} + (\Phi_{z_1}, \Phi_{z_2}, 6\Phi_X, 6\Phi_Y) \\
 SO(16) &\rightarrow SU(9)_{GUT} + (G_G, \Phi_{F_3}, 12\Phi_w, 8\Phi_V) \\
 SO(20) &\rightarrow SU(11)_{GUT} + (\Phi_{F_8}, \Phi_{F_{15}}, 12\Phi_T, 16\Phi_U) \\
 SO(24) &\rightarrow SU(13)_{GUT} + (G_{F_3}, G_{F_8}, G_{F_{15}}, 5\Phi_F, \Phi_{Q_3}, \Phi_{Q_8}, 4\Phi_U, 12\Phi_R, 12\Phi_S)
 \end{aligned} \tag{6}$$

Legend for Chapter 3: Same as Table 3 plus: H_L = Standard Model Higgs; Φ_Z, Φ_W = Longitudinal Polarizations (LP’s) for Standard Weak IVB’s, Φ_X, Φ_Y = LP’s for Georgi-Glashow Leptoquarks; $\Phi_{z_{1,2}}$ = LP’s for Hyperflavor-Weak (HFW) diagonal IVB’s; Φ_w = LP’s for HFW off-diagonal charged and neutral IVB’s; Φ_V = LP’s for $SU(7)$ Gravi-Hyperflavor “Bosons”; G_G = Gravitational Goldstone; Φ_F = LP’s for WIMP-Gravitons; $\Phi_R, \Phi_S, \Phi_T, \Phi_U$ = LP’s for off-diagonal $SU(11)$ Grand Bosons; G_F = WIMP-Gravitational Goldstones; Φ_Q = LP’s for Generators.

Combining Postulates Two, Three and Four, we are led to the following Boson GUT examples:

Example 1 – $SU(5) + SO(8) \rightarrow F4$, A 52-plet Boson GUT including 4 extra Scalars:

$$\left(\begin{array}{cccc} g^{3\wedge} & g^{8\wedge} & \gamma^\wedge & Z^\wedge \\ g^{r\bar{g}\wedge} & g^{g\bar{r}\wedge} & X^{\bar{r}-\wedge} & Y^{\bar{r}+\wedge} \\ g^{r\bar{b}\wedge} & g^{b\bar{r}\wedge} & X^{\bar{g}-\wedge} & Y^{\bar{g}+\wedge} \\ g^{g\bar{b}\wedge} & g^{b\bar{g}\wedge} & X^{\bar{b}-\wedge} & Y^{\bar{b}+\wedge} \\ X^{r+\wedge} & X^{g+\wedge} & X^{b+\wedge} & W^{-\wedge} \\ Y^{r-\wedge} & Y^{g-\wedge} & Y^{b-\wedge} & W^{+\wedge} \end{array} \right) + \left(\begin{array}{cccc} g^{3\vee} & g^{8\vee} & \gamma^\vee & Z^\vee \\ g^{r\bar{g}\vee} & g^{g\bar{r}\vee} & X^{\bar{r}-\vee} & Y^{\bar{r}+\vee} \\ g^{r\bar{b}\vee} & g^{b\bar{r}\vee} & X^{\bar{g}-\vee} & Y^{\bar{g}+\vee} \\ g^{g\bar{b}\vee} & g^{b\bar{g}\vee} & X^{\bar{b}-\vee} & Y^{\bar{b}+\vee} \\ X^{r+\vee} & X^{g+\vee} & X^{b+\vee} & W^{-\vee} \\ Y^{r-\vee} & Y^{g-\vee} & Y^{b-\vee} & W^{+\vee} \\ H_L & \Phi_Z & \Phi_{W^-} & \Phi_{W^+} \end{array} \right) \rightarrow F4 \quad (7)$$

Example 2 – $SU(7) + SO(12) \rightarrow E6' - G2_R$, A 114-plet Boson GUT including 18 extra Scalars:

$$\left(\begin{array}{cccccc} g^{3\wedge} & g^{8\wedge} & \gamma^\wedge & Z^\wedge & z_1^\wedge & z_2^\wedge \\ g^\wedge & g^\wedge & X^\wedge & Y^\wedge & V^\wedge & V^\wedge \\ g^\wedge & g^\wedge & X^\wedge & Y^\wedge & V^\wedge & V^\wedge \\ g^\wedge & g^\wedge & X^\wedge & Y^\wedge & w_1^{+\wedge} & w_1'^{+\wedge} \\ X^\wedge & X^\wedge & X^\wedge & W^\wedge & w_2^{+\wedge} & w_2'^{+\wedge} \\ Y^\wedge & Y^\wedge & Y^\wedge & W^\wedge & w^{0*\wedge} & w'^{0*\wedge} \\ V^\wedge & V^\wedge & w_1^{-\wedge} & w_2^{-\wedge} & w^{0\wedge} & H_H \\ V^\wedge & V^\wedge & w_1'^{-\wedge} & w_2'^{-\wedge} & w'^{0\wedge} & H_P \end{array} \right) + \left(\begin{array}{cccccc} g^{3\vee} & g^{8\vee} & \gamma^\vee & Z^\vee & z_1^\vee & z_2^\vee \\ g^\vee & g^\vee & X^\vee & Y^\vee & V^\vee & V^\vee \\ g^\vee & g^\vee & X^\vee & Y^\vee & V^\vee & V^\vee \\ g^\vee & g^\vee & X^\vee & Y^\vee & w_1^{+\vee} & w_1'^{+\vee} \\ X^\vee & X^\vee & X^\vee & W^\vee & w_2^{+\vee} & w_2'^{+\vee} \\ Y^\vee & Y^\vee & Y^\vee & W^\vee & w^{0*\vee} & w'^{0*\vee} \\ V^\vee & V^\vee & w_1^{-\vee} & w_2^{-\vee} & w^{0\vee} & H^- \\ V^\vee & V^\vee & w_1'^{-\vee} & w_2'^{-\vee} & w'^{0\vee} & H^+ \\ H_L & \Phi_Z & \Phi_{W^-} & \Phi_{W^+} & \Phi_{z_1} & \Phi_{z_2} \\ \Phi_{Y^{r-}} & \Phi_{Y^{g-}} & \Phi_{Y^{b-}} & \Phi_{Y^{\bar{r}+}} & \Phi_{Y^{\bar{g}+}} & \Phi_{Y^{\bar{b}+}} \\ \Phi_{X^{r+}} & \Phi_{X^{g+}} & \Phi_{X^{b+}} & \Phi_{X^{\bar{r}-}} & \Phi_{X^{\bar{g}-}} & \Phi_{X^{\bar{b}-}} \end{array} \right) \quad (8)$$

Example 3 – $SU(9) + SO(16) \rightarrow E8 - F4_R$, A 200-plet Boson GUT including 40 extra Scalars:

$$\left(\begin{array}{cccccc} g^{3\wedge} & g^{8\wedge} & \gamma^\wedge & Z^\wedge & z_1^\wedge & z_2^\wedge & G^\wedge & F_3^\wedge \\ g^\wedge & g^\wedge & X^\wedge & Y^\wedge & V^\wedge & V^\wedge & T^\wedge & T^\wedge \\ g^\wedge & g^\wedge & X^\wedge & Y^\wedge & V^\wedge & V^\wedge & T^\wedge & T^\wedge \\ g^\wedge & g^\wedge & X^\wedge & Y^\wedge & w^\wedge & w^\wedge & T^\wedge & T^\wedge \\ X^\wedge & X^\wedge & X^\wedge & W^\wedge & w^\wedge & w^\wedge & U^\wedge & U^\wedge \\ Y^\wedge & Y^\wedge & Y^\wedge & W^\wedge & w^\wedge & w^\wedge & U^\wedge & U^\wedge \\ V^\wedge & V^\wedge & w^\wedge & w^\wedge & w^\wedge & H_H & U^\wedge & U^\wedge \\ V^\wedge & V^\wedge & w^\wedge & w^\wedge & w^\wedge & H_P & U^\wedge & U^\wedge \\ T^\wedge & T^\wedge & T^\wedge & U^\wedge & U^\wedge & U^\wedge & U^\wedge & F_1^\wedge \\ T^\wedge & T^\wedge & T^\wedge & U^\wedge & U^\wedge & U^\wedge & U^\wedge & F_2^\wedge \end{array} \right) + \left(\begin{array}{cccccc} g^{3\vee} & g^{8\vee} & \gamma^\vee & Z^\vee & z_1^\vee & z_2^\vee & G^\vee & F_3^\vee \\ g^\vee & g^\vee & X^\vee & Y^\vee & V^\vee & V^\vee & T^\vee & T^\vee \\ g^\vee & g^\vee & X^\vee & Y^\vee & V^\vee & V^\vee & T^\vee & T^\vee \\ g^\vee & g^\vee & X^\vee & Y^\vee & w^\vee & w^\vee & T^\vee & T^\vee \\ X^\vee & X^\vee & X^\vee & W^\vee & w^\vee & w^\vee & U^\vee & U^\vee \\ Y^\vee & Y^\vee & Y^\vee & W^\vee & w^\vee & w^\vee & U^\vee & U^\vee \\ V^\vee & V^\vee & w^\vee & w^\vee & w^\vee & H^- & U^\vee & U^\vee \\ V^\vee & V^\vee & w^\vee & w^\vee & w^\vee & H^+ & U^\vee & U^\vee \\ T^\vee & T^\vee & T^\vee & U^\vee & U^\vee & U^\vee & U^\vee & F_1^\vee \\ T^\vee & T^\vee & T^\vee & U^\vee & U^\vee & U^\vee & U^\vee & F_2^\vee \\ H_L & \Phi_Z & \Phi_W & \Phi_W & \Phi_{z_1} & \Phi_{z_2} & G_G & \Phi_{F_3,0} \\ \Phi_Y & \Phi_Y & \Phi_Y & \Phi_Y & \Phi_Y & \Phi_Y & \Phi_{w^0} & \Phi_{w^0} \\ \Phi_X & \Phi_X & \Phi_X & \Phi_X & \Phi_X & \Phi_X & \Phi_{w^0} & \Phi_{w^0} \\ \Phi_{W^-} & \Phi_{W^-} & \Phi_{W^-} & \Phi_{W^-} & \Phi_{W^+} & \Phi_{W^+} & \Phi_{W^+} & \Phi_{W^+} \\ \Phi_V & \Phi_V & \Phi_V & \Phi_V & \Phi_V & \Phi_V & \Phi_V & \Phi_V \end{array} \right) \quad (9)$$

Example 4 – $SU(11) + SO(20) \rightarrow E_{10} - E_{6R}'$, A 310-plet Boson GUT (Equation 10):

$$\begin{pmatrix}
 g^{3\wedge} & g^{8\wedge} & \gamma^\wedge & Z^\wedge & z_1^\wedge & z_2^\wedge & G^\wedge & F_3^\wedge & F_8^\wedge & F_{15}^\wedge \\
 g^\wedge & g^\wedge & X^\wedge & Y^\wedge & V^\wedge & V^\wedge & T^\wedge & T^\wedge & R^\wedge & R^\wedge \\
 g^\wedge & g^\wedge & X^\wedge & Y^\wedge & V^\wedge & V^\wedge & T^\wedge & T^\wedge & R^\wedge & R^\wedge \\
 g^\wedge & g^\wedge & X^\wedge & Y^\wedge & w^\wedge & w^\wedge & T^\wedge & T^\wedge & R^\wedge & R^\wedge \\
 X^\wedge & X^\wedge & X^\wedge & W^\wedge & w^\wedge & w^\wedge & U^\wedge & U^\wedge & S^\wedge & S^\wedge \\
 Y^\wedge & Y^\wedge & Y^\wedge & W^\wedge & w^\wedge & w^\wedge & U^\wedge & U^\wedge & S^\wedge & S^\wedge \\
 V^\wedge & V^\wedge & w^\wedge & w^\wedge & w^\wedge & H_H & U^\wedge & U^\wedge & S^\wedge & S^\wedge \\
 V^\wedge & V^\wedge & w^\wedge & w^\wedge & w^\wedge & H_p & U^\wedge & U^\wedge & U^\wedge & U^\wedge \\
 T^\wedge & T^\wedge & T^\wedge & U^\wedge & U^\wedge & U^\wedge & U^\wedge & F_1^\wedge & F_B^\wedge & F_D^\wedge \\
 T^\wedge & T^\wedge & T^\wedge & U^\wedge & U^\wedge & U^\wedge & U^\wedge & F_2^\wedge & F_B^{*\wedge} & F_D^{*\wedge} \\
 R^\wedge & R^\wedge & R^\wedge & S^\wedge & S^\wedge & S^\wedge & U^\wedge & F_A^\wedge & F_C^\wedge & F_E^\wedge \\
 R^\wedge & R^\wedge & R^\wedge & S^\wedge & S^\wedge & S^\wedge & U^\wedge & F_A^{*\wedge} & F_C^{*\wedge} & F_E^{*\wedge}
 \end{pmatrix}
 +
 \begin{pmatrix}
 g^{3\vee} & g^{8\vee} & \gamma^\vee & Z^\vee & z_1^\vee & z_2^\vee & G^\vee & F_3^\vee & F_8^\vee & F_{15}^\vee \\
 g^\vee & g^\vee & X^\vee & Y^\vee & V^\vee & V^\vee & T^\vee & T^\vee & R^\vee & R^\vee \\
 g^\vee & g^\vee & X^\vee & Y^\vee & V^\vee & V^\vee & T^\vee & T^\vee & R^\vee & R^\vee \\
 g^\vee & g^\vee & X^\vee & Y^\vee & w^\vee & w^\vee & T^\vee & T^\vee & R^\vee & R^\vee \\
 X^\vee & X^\vee & X^\vee & W^\vee & w^\vee & w^\vee & U^\vee & U^\vee & S^\vee & S^\vee \\
 Y^\vee & Y^\vee & Y^\vee & W^\vee & w^\vee & w^\vee & U^\vee & U^\vee & S^\vee & S^\vee \\
 V^\vee & V^\vee & w^\vee & w^\vee & w^\vee & H^- & U^\vee & U^\vee & S^\vee & S^\vee \\
 V^\vee & V^\vee & w^\vee & w^\vee & w^\vee & H^+ & U^\vee & U^\vee & U^\vee & U^\vee \\
 T^\vee & T^\vee & T^\vee & U^\vee & U^\vee & U^\vee & U^\vee & F_1^\vee & F_B^\vee & F_D^\vee \\
 T^\vee & T^\vee & T^\vee & U^\vee & U^\vee & U^\vee & U^\vee & F_2^\vee & F_B^{*\vee} & F_D^{*\vee} \\
 R^\vee & R^\vee & R^\vee & S^\vee & S^\vee & S^\vee & U^\vee & F_A^\vee & F_C^\vee & F_E^\vee \\
 R^\vee & R^\vee & R^\vee & S^\vee & S^\vee & S^\vee & U^\vee & F_A^{*\vee} & F_C^{*\vee} & F_E^{*\vee} \\
 H_L & \Phi_Z & \Phi_W & \Phi_W & \Phi_{z_1} & \Phi_{z_2} & G_G & \Phi_{F_3,0} & \Phi_{F_8,0} & \Phi_{F_{15},0} \\
 \Phi_Y & \Phi_Y & \Phi_Y & \Phi_Y & \Phi_Y & \Phi_Y & \Phi_w & \Phi_w & \Phi_T & \Phi_T \\
 \Phi_X & \Phi_X & \Phi_X & \Phi_X & \Phi_X & \Phi_X & \Phi_w & \Phi_w & \Phi_T & \Phi_T \\
 \Phi_w & \Phi_w & \Phi_w & \Phi_w & \Phi_w & \Phi_w & \Phi_w & \Phi_w & \Phi_T & \Phi_T \\
 \Phi_V & \Phi_V & \Phi_V & \Phi_V & \Phi_V & \Phi_V & \Phi_V & \Phi_V & \Phi_T & \Phi_T \\
 \Phi_U & \Phi_U & \Phi_U & \Phi_U & \Phi_U & \Phi_U & \Phi_U & \Phi_U & \Phi_T & \Phi_T \\
 \Phi_U & \Phi_U & \Phi_U & \Phi_U & \Phi_U & \Phi_U & \Phi_U & \Phi_U & \Phi_T & \Phi_T
 \end{pmatrix}$$

Combining $SU(N+1)$ algebras with $SO(2N)$ algebras allows us to represent spin-up bosons (the $SU(N+1)$ degrees of freedom (dgf's)) plus spin-down bosons, scalar/ Higgs bosons, and the longitudinal degrees of freedom Φ that allow bosons of mass and spin (the $SO(2N)$ dgf's). Table 4 reviews the bosons in an $SO(24)$ Boson-Higgs-Longitudinal GUT. Note that the $SU(N+1)$ Boson GUT's are on the left bottom corner of Table 4 (with basis bosons across the top), whereas the Higgs-Longitudinal GUT's are on the right top corner of the Table.

The size difference between the $SO(2N)$ algebras and the $SU(N+1)$ algebras seems to determine the number of Higgs and longitudinal degrees of freedom (dgf's) unambiguously. Note that the Gravity – WIMP-Gravity sector seems to divide into $(F_1, F_2, F_3, F_8, F_{15})$ times three constrained (via Clifford bivectors and/ or BRST Theory) spin projections each, either $(+2,0,-2)$ or $(+1,0,-1)$; and (F_{A-E}) times five spin projections each: $(+2,+1,0,-1,-2)$.

Next, we will study simplices as particle multiplets, then we will analyze important sub-components common to all three of these groups: 1) a 3-simplex of Electro-Color, 2) two related versions of Gravi-Weak: a 3-simplex of Gravi-Pati-Salam-Weak and a 4-simplex of Gravi-Hyperflavor-Weak and 3) a 2-simplex of Generations. The product of these simplices builds lattices representative of Exceptional groups.

Table 4 – An $SO(24)$ Boson GUT with Higgs/ Scalars and Longitudinal Dgf's

12-D $SO(24)$											
10-D $SO(20)$											
8-D $SO(16)$											
6-D $SO(12)$											
4-D $SO(8)$											
g^3	g^8	γ	Z			z_1	z_2			G	F_3
W_1^-	W_1^+	H	Φ_Z	Φ_{z_1}	Φ_{z_2}	G_G	$\Phi_{F_3,0}$	$\Phi_{F_8,0}$	$\Phi_{F_{15},0}$	G_{F_3}	G_{F_8}
$g^{r\bar{g}}$	$g^{r\bar{b}}$	$g^{g\bar{b}}$	Φ_{W_1}	Φ_{X^r}	Φ_{Y^r}	Φ_{w^0}	Φ_{w^0}	Φ_{T_1}	$\Phi_{T_1}^*$	$G_{F_{15}}$	Φ_{F_A}
$g^{g\bar{r}}$	$g^{b\bar{r}}$	$g^{b\bar{g}}$	$\Phi_{W_1}^*$	Φ_{X^g}	Φ_{Y^g}	Φ_{w^0}	Φ_{w^0}	Φ_{T_2}	$\Phi_{T_2}^*$	Φ_{F_B}	Φ_{F_C}
X^{r+}	$X^{\bar{r}-}$	Y^{r-}	$Y^{\bar{r}+}$	Φ_{X^b}	Φ_{Y^b}	Φ_{w^-}	Φ_{w^-}	Φ_{T_3}	$\Phi_{T_3}^*$	Φ_{F_D}	Φ_{F_E}
X^{g+}	$X^{\bar{g}-}$	Y^{g-}	$Y^{\bar{g}+}$	$\Phi_{X^r}^*$	$\Phi_{Y^r}^*$	Φ_{w^-}	Φ_{w^-}	Φ_{T_4}	$\Phi_{T_4}^*$	Φ_{Q_3}	Φ_{Q_8}
X^{b+}	$X^{\bar{b}-}$	Y^{b-}	$Y^{\bar{b}+}$	$\Phi_{X^g}^*$	$\Phi_{Y^g}^*$	Φ_{w^+}	Φ_{w^+}	Φ_{T_5}	$\Phi_{T_5}^*$	Φ_{U_8}	$\Phi_{U_8}^*$
V_L^{A+}	V_L^{A-}	w_1^-	$w_1'^-$	$\Phi_{X^b}^*$	$\Phi_{Y^b}^*$	Φ_{w^+}	Φ_{w^+}	Φ_{T_6}	$\Phi_{T_6}^*$	Φ_{U_9}	$\Phi_{U_9}^*$
V_L^{V+}	V_L^{V-}	w_2^-	$w_2'^-$	H^-	H^+	Φ_V	Φ_V	Φ_{U_1}	$\Phi_{U_1}^*$	Φ_{S_1}	$\Phi_{S_1}^*$
V_R^{A+}	V_R^{A-}	w_1^+	$w_1'^+$	w_1^0	w_1^{0*}	Φ_V	Φ_V	Φ_{U_2}	$\Phi_{U_2}^*$	Φ_{S_2}	$\Phi_{S_2}^*$
V_R^{V+}	V_R^{V-}	w_2^+	$w_2'^+$	w_1^0	w_1^{0*}	Φ_V	Φ_V	Φ_{U_3}	$\Phi_{U_3}^*$	Φ_{S_3}	$\Phi_{S_3}^*$
T_1^0	T_1^{0*}	T_2^{0*}	T_4^0	T_4^{0*}	T_5^{0*}	Φ_V	Φ_V	Φ_{U_4}	$\Phi_{U_4}^*$	Φ_{S_4}	$\Phi_{S_4}^*$
T_2^0	T_3^0	T_3^{0*}	T_5^0	T_6^0	T_6^{0*}	F_1	F_2	Φ_{U_5}	$\Phi_{U_5}^*$	Φ_{S_5}	$\Phi_{S_5}^*$
$U_1^{\bar{c}+}$	$U_2^{\bar{c}+}$	$U_3^{\bar{c}+}$	$U_4^{\bar{c}+}$	$U_5^{\bar{c}+}$	$U_6^{\bar{c}+}$	$U_7^{\bar{c}+}$	U_{10}^{c+}	Φ_{U_6}	$\Phi_{U_6}^*$	Φ_{S_6}	$\Phi_{S_6}^*$
U_1^{c-}	U_2^{c-}	U_3^{c-}	U_4^{c-}	U_5^{c-}	U_6^{c-}	U_7^{c-}	$U_{10}^{\bar{c}-}$	Φ_{U_7}	$\Phi_{U_7}^*$	Φ_{R_1}	$\Phi_{R_1}^*$
R_1^+	R_4^+	R_1^-	R_4^-	S_1^{c+}	S_4^{c+}	$S_1^{\bar{c}-}$	$S_4^{\bar{c}-}$	$\Phi_{U_{10}}$	$\Phi_{U_{10}}^*$	Φ_{R_2}	$\Phi_{R_2}^*$
R_2^+	R_5^{c+}	R_2^-	R_5^-	S_2^{c+}	S_5^{c+}	$S_2^{\bar{c}-}$	$S_5^{\bar{c}-}$	U_8^{c-}	$U_8^{\bar{c}+}$	Φ_{R_3}	$\Phi_{R_3}^*$
R_3^+	R_6^{c+}	R_3^-	R_6^-	S_3^{c+}	S_6^{c+}	$S_3^{\bar{c}-}$	$S_6^{\bar{c}-}$	U_9^{c-}	$U_9^{\bar{c}+}$	Φ_{R_4}	$\Phi_{R_4}^*$
F_A	F_B	F_C	F_D	F_E	F_A^*	F_B^*	F_C^*	F_D^*	F_E^*	Φ_{R_5}	$\Phi_{R_5}^*$
P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	Φ_{R_6}	$\Phi_{R_6}^*$
P_1^*	P_2^*	P_3^*	P_4^*	P_5^*	P_6^*	P_7^*	P_8^*	P_9^*	P_{10}^*	Q_1	Q_2
N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8	N_9	N_{10}	Q_4	Q_5
N_1^*	N_2^*	N_3^*	N_4^*	N_5^*	N_6^*	N_7^*	N_8^*	N_9^*	N_{10}^*	Q_6	Q_7

4. Simplices as Particle Multiplets

Postulate 5 – Because Simplices and Particle Multiplets share common mathematical properties, it is assumed that Particle Multiplets may be represented by Simplices.

Consider the example of a 3-simplex (tetrahedron) as a particle multiplet. This is the simplest example that demonstrates all of the basic properties of these simplices, but is also justified by Hyperflavor-Electroweak (HEW) [4] and examples to follow in this paper. We will assume an $SU(4)$ Lie algebra with diagonal operators (C_3, C_8, C_{15}) .

We want to construct a simplex with the following properties: 1) the sum of all charges within a particle multiplet equals zero, and 2) all particles have the same distance from each other. As a consequence of these two requirements, we realize that all particles must also have the same radius about the origin.

Table 5 is deduced by process of trial and error. Note that the strengths of the charges (C_3, C_8, C_{15}) are introduced in a ratio of $(1, \sqrt{3}, \sqrt{6})$. In the general case, this approaches a ratio of the square root of the progression of Special Orthogonal orders $(1, \sqrt{3}, \sqrt{6}, \sqrt{10}, \sqrt{15}, \dots, \sqrt{n(n+1)/2})$. Coincidentally, if we generalize the Gell-Mann matrices to n dimensions, then the renormalization pattern for those basis matrices $(\lambda_3, \lambda_8, \lambda_{15}, \lambda_{24}, \lambda_{35}, \dots, \lambda_{n(n+2)})$ follows the identical pattern. These four particle vectors (A, B, C, D) exist in a three-dimensional space (C_3, C_8, C_{15}) , are each one unit from each other (such as $\Delta_{AB}^2 = (\frac{1}{2} - \frac{-1}{2})^2 + \frac{0^2}{3} + \frac{0^2}{6} = 1$, $\Delta_{AD}^2 = (\frac{1}{2} - 0)^2 + \frac{1}{3}(\frac{1}{2} - 0)^2 + \frac{1}{6}(\frac{-1}{2} - \frac{3}{2})^2 = 1$, etc.) and are each $\sqrt{\frac{3}{8}}$'s of a unit from the origin (such as $r_A^2 = r_B^2 = (\pm \frac{1}{2})^2 + \frac{1}{3}(\frac{1}{2})^2 + \frac{1}{6}(\frac{-1}{2})^2 = \frac{3}{8}$, $r_C^2 = (0)^2 + \frac{1}{3}(-1)^2 + \frac{1}{6}(\frac{-1}{2})^2 = \frac{3}{8}$, and $r_D^2 = (0)^2 + \frac{1}{3}(0)^2 + \frac{1}{6}(\frac{3}{2})^2 = \frac{3}{8}$). In the general case, our n -simplex will exist in n -dimensions; have $(n+1)$ particle vectors that are one unit from each other, and $\sqrt{n/(2n+2)}$ of a unit from the origin. Note, that by construction, we have

$$\sum_{A,B,C,D} C_3 = \frac{1}{2} + \frac{-1}{2} + 0 = 0, \quad \sum_{A,B,C,D} \sqrt{3}C_8 = \frac{1}{2} + \frac{1}{2} - 1 + 0 = 0, \quad \text{and} \quad \sum_{A,B,C,D} \sqrt{6}C_{15} = \frac{-1}{2} + \frac{-1}{2} + \frac{-1}{2} + \frac{3}{2} = 0.$$

Table 5 – A 4 Particle Multiplet as a 3-Simplex

Charges → ↓ Fermions	C_3	$\sqrt{3}C_8$	$\sqrt{6}C_{15}$	C'_8	Charges → ↓ Fermions	C_3	$\sqrt{3}C_8$	$\sqrt{6}C_{15}$	C'_8
A	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	\bar{A}	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0
B	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	\bar{B}	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0
C	0	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	\bar{C}	0	1	$\frac{1}{2}$	$\frac{1}{2}$
D	0	0	$\frac{3}{2}$	$\frac{1}{2}$	\bar{D}	0	0	$-\frac{3}{2}$	$-\frac{1}{2}$
Sum	0	0	0	0	Sum	0	0	0	0

Grand Unified Theories (GUT's) generally require this feature within a particle multiplet. Interestingly enough, a secondary conserved quantum number emerges from the mathematics: $C'_8 = (\sqrt{3}C_8 + \sqrt{6}C_{15})/3 = 0, \pm \frac{1}{2}, \pm 1$, etc. This is due to the fact that both charges have a common factor of $\sqrt{3}$, and has a net effect of collapsing the algebra down into one fewer dimensions and introducing a broken symmetry. In the general case, we will have more secondary conserved quantum numbers, such as $C'_{24} = (\sqrt{10}C_{24} + \sqrt{15}C_{35})/5 = 0, \pm \frac{1}{2}, \pm 1$, etc., and so on. These geometrical constraints may be related to Clifford bivectors and the first-class constraints of BRST formalism (Becchi, Rouet, Stora and Tyutin) [8].

Note that antiparticles could simply be the inversion operator applied to these particle states, such that $\bar{A} = (-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2})$, etc. (see Table 5). This 3-simplex (A, B, C, D) and its dual $(\bar{A}, \bar{B}, \bar{C}, \bar{D})$ collectively comprise the eight vertices of a cube. We may rotate from our $SU(4)$ basis (C_3, C_8, C_{15}) into a cubic basis (C_x, C_y, C_z) and vice versa with the transformations:

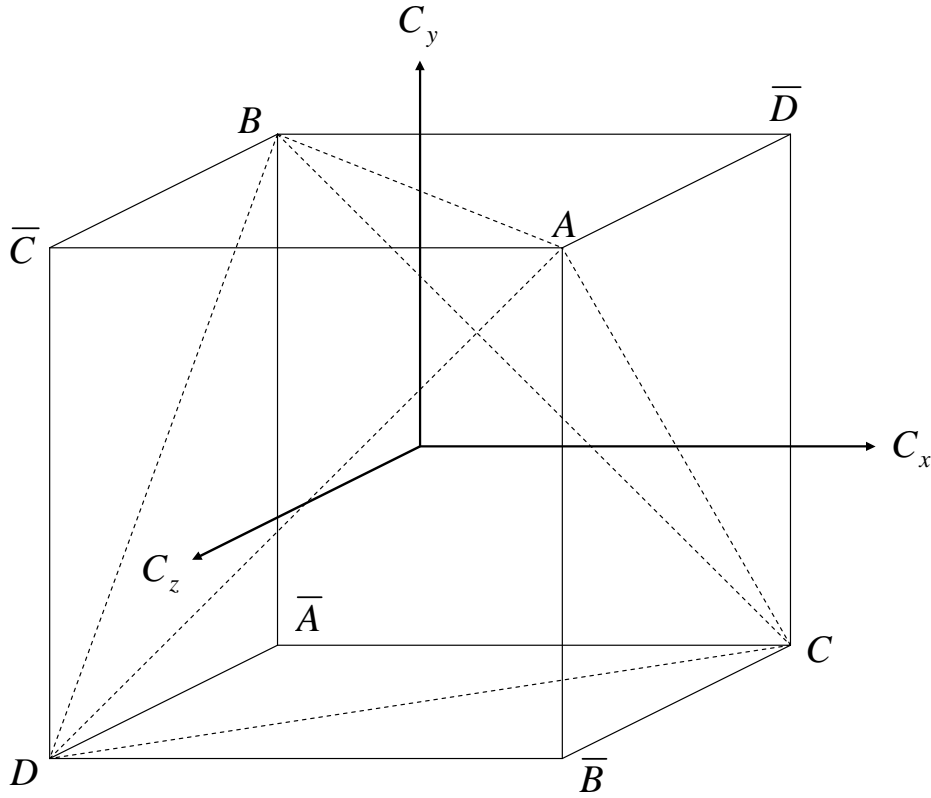
$$\begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix} = \frac{1}{3\sqrt{2}} \begin{pmatrix} 3 & -\sqrt{3} & -\sqrt{6} \\ 0 & 2\sqrt{3} & -\sqrt{6} \\ 3 & \sqrt{3} & \sqrt{6} \end{pmatrix} \begin{pmatrix} C_3 \\ C_8 \\ C_{15} \end{pmatrix} \quad \& \quad \begin{pmatrix} C_3 \\ C_8 \\ C_{15} \end{pmatrix} = \frac{1}{2\sqrt{3}} \begin{pmatrix} \sqrt{6} & 0 & \sqrt{6} \\ -\sqrt{2} & 2\sqrt{2} & \sqrt{2} \\ -2 & -2 & 2 \end{pmatrix} \begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix} \quad (11)$$

such that $A = \frac{1}{2\sqrt{2}}(1,1,1)$, $B = \frac{1}{2\sqrt{2}}(-1,1,-1)$, $C = \frac{1}{2\sqrt{2}}(1,-1,-1)$, $D = \frac{1}{2\sqrt{2}}(-1,-1,1)$, $\bar{A} = \frac{-1}{2\sqrt{2}}(1,1,1)$, $\bar{B} = \frac{1}{2\sqrt{2}}(1,-1,1)$, $\bar{C} = \frac{1}{2\sqrt{2}}(-1,1,1)$ and $\bar{D} = \frac{1}{2\sqrt{2}}(1,1,-1)$ in the cubic (C_x, C_y, C_z) basis (Figure 2).

In the vector notation of both bases, we have for example:

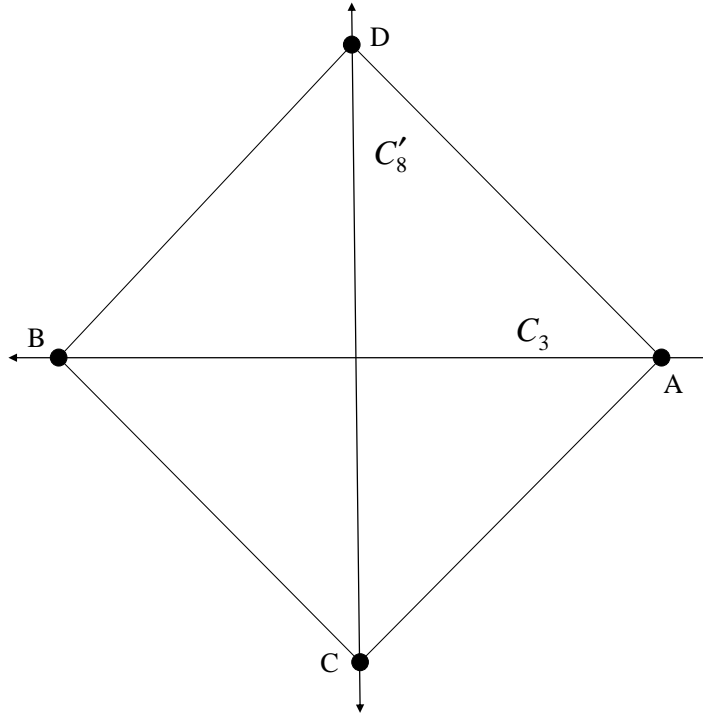
$$A = \frac{1}{2}\hat{C}_3 + \frac{1}{2\sqrt{3}}\hat{C}_8 - \frac{1}{2\sqrt{6}}\hat{C}_{15} = \frac{1}{2\sqrt{2}}\hat{C}_x + \frac{1}{2\sqrt{2}}\hat{C}_y + \frac{1}{2\sqrt{2}}\hat{C}_z.$$

Figure 2 – Nested Dual Tetrahedra (3-Simplices) as a Cube



How do these simplices relate to $SU(n+1)$ or $SO(2n)$ Lie algebras? Figure 3 is a Petrie polygon of this 3-simplex using the basis vectors of the collapsed algebra (C_3, C'_8) . This figure demonstrates how the four particle vectors of this 3-simplex example can be interconnected with the six vector operators (such as AB, AC, AD, BC, BD, CD) of a rank-2 $SO(4)$ (this defines direction in the original basis (C_3, C_8, C_{15}) , but not to and from). If we require two different operators for each direction (this also defines to and from), then we have the twelve operators of a rank- $(2+2)$ $SO(4) \times SO(4)$. And adding in the three basis vectors (C_3, C_8, C_{15}) gives the fifteen operators of a rank-3 $SU(4)$. In the general case, an n -simplex has $(n+1)$ particle vectors that can be interconnected with an $SO(n+1)$ of order $[n(n+1)]/2$. Doubling this order and adding n basis vectors, we have an order of $n(n+2)$, which equates with an $SU(n+1)$ algebra. Previously, the author [4] assumed a relationship between the rank of an $SU(n+1)$ Special Unitary Lie algebra and the number of dimensions in which it exists. This example relates the rank of these Special Unitary Lie algebras specifically to an n -dimensional n -simplex.

Figure 3 – Petrie Polygon of a 3-Simplex



4.1 A 3-Simplex of Electro-Color

A relevant application of the 3-simplex is Electro-Color. Quantum Chromodynamics or Color Theory is a 2-simplex (equilateral triangle) when only quarks are considered, and Lisi [1] and the author [5] have described a triangular lattice of $G2_C \rightarrow SU(3)_C \times SO(4)_q \rightarrow SU(3)_C \times SO(3,1)_{q+\bar{q}} \rightarrow SU(3)_C + 3_q + \bar{3}_{\bar{q}}$ color somewhat thoroughly. However, if we consider the possibility that leptons are a color singlet (the “color” white), and part of the Electro-Color tetrahedron (the triangle of (r, g, b) color is one of the four faces of this tetrahedron), then we are lead to the results in Table 6. The color charges follow Lisi’s definitions, and the definition of white is uniquely determined by the secondary conserved quantum number $g'_8 = (\sqrt{3} g_8 - \frac{1}{2} Y')/3$, the sum of charges (such as $\sum_{r,g,b,w} g_3 = 0$ etc.), and the definition of Y' .

Table 7 presents gluons as color translation vectors, with $q_i^{c2} = g^{c2\bar{c1}} q_i^{c1}$ and leads to Postulate 6.

Postulate 6 – Bosons are defined in terms of translation vectors between Fermions. Therefore Fermions and Bosons exist in Reciprocal (Dual) lattices from each other.

Table 6 – Two Dual 3-Simplices of Electro-Color

Dim's	aa	ab	ac	
Bosons	g_3	g_8	γ	
Charges → ↓ Fermions	g_3	$\sqrt{3}g_8$	$-\frac{3}{2}Y'$	g'_8
Red r	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0
Green g	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0
Blue b	0	-1	$-\frac{1}{2}$	$-\frac{1}{2}$
White w	0	0	$\frac{3}{2}$	$\frac{1}{2}$

Dim's	aa	ab	ac	
Bosons	g_3	g_8	γ	
Charges → ↓ Fermions	g_3	$\sqrt{3}g_8$	$-\frac{3}{2}Y'$	g'_8
Anti-Red \bar{r}	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0
Anti-Green \bar{g}	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0
Anti-Blue \bar{b}	0	1	$\frac{1}{2}$	$\frac{1}{2}$
Anti-White \bar{w}	0	0	$-\frac{3}{2}$	$-\frac{1}{2}$

Table 7 – The Off-Diagonal Gluons as Translation Vectors

Charges → ↓ Bosons	Δg_3	$\sqrt{3} \times \Delta g_8$	$-\frac{3}{2} \times \Delta Y'$	$\Delta g'_8$
$g^{r\bar{g}}$	1	0	0	0
$g^{r\bar{b}}$	$\frac{1}{2}$	$\frac{3}{2}$	0	$\frac{1}{2}$
$g^{g\bar{b}}$	$-\frac{1}{2}$	$\frac{3}{2}$	0	$\frac{1}{2}$

Charges → ↓ Bosons	Δg_3	$\sqrt{3} \times \Delta g_8$	$-\frac{3}{2} \times \Delta Y'$	$\Delta g'_8$
$g^{g\bar{r}}$	-1	0	0	0
$g^{b\bar{r}}$	$-\frac{1}{2}$	$-\frac{3}{2}$	0	$-\frac{1}{2}$
$g^{b\bar{g}}$	$\frac{1}{2}$	$-\frac{3}{2}$	0	$-\frac{1}{2}$

This definition of bosons and fermions leads to the conclusion that all fermion-boson-fermion vertices are simple three-legged vertices (Section 6.3). New color combinations such as $r\bar{w}, g\bar{w}, b\bar{w}, w\bar{r}, w\bar{g}$ and $w\bar{b}$ will be important to the X and Y Leptoquark bosons of $SU(5)$ (Sections 4.2 and 6.3). The four "colors" (r, g, b, w) are the four corners of this 3-simplex. This $SU(3) \times U(1)$ product of Lie algebras has three charges $(g_3, \sqrt{3}g_8, -\frac{3}{2}Y')$. From Table 5, we might expect $SU(4)$ charges of $(g_3, \sqrt{3}g_8, \sqrt{6}g_{15})$, which implies the GUT result:

$$\sin^2 \theta_w = \left(\frac{g_{15}}{Y'} \right)^2 = \left(\frac{-3}{2\sqrt{6}} \right)^2 = \frac{3}{8} \quad (12)$$

Y' is defined as Y_L for $(u_L, e_L, d_L, \nu_{eL})$ or Y_R for $(u_R, e_R, d_R, \nu_{eR})$, & may be summarized as:

$$Y' = \begin{cases} \frac{1}{3} & \text{for quarks } (u, d, c, s, t, b) \text{ or color } = (r, g, b) \\ -1 & \text{for leptons } (e, \nu_e, \mu, \nu_\mu, \tau, \nu_\tau) \text{ or color } = w \\ -\frac{1}{3} & \text{for anti-quarks } (\bar{u}, \bar{d}, \bar{c}, \bar{s}, \bar{t}, \bar{b}) \text{ or color } = (\bar{r}, \bar{g}, \bar{b}) \\ 1 & \text{for anti-leptons } (\bar{e}, \bar{\nu}_e, \bar{\mu}, \bar{\nu}_\mu, \bar{\tau}, \bar{\nu}_\tau) \text{ or color } = \bar{w} \end{cases} \quad (13)$$

4.2 Gravi-Weak

Lisi's *E8* TOE used a Gravi-Weak unification of a Pati-Salam Weak force and a Gravitational force based on Clifford algebra. At first glance, this seems to be as odd of a pairing of forces as Electro-Color, but these strange relationships may yield clues to the structure of Spacetime and Hyperspace, and will be discussed later.

Lisi's Gravi-Pati-Salam-Weak had two Weak charges and two Gravitational charges. In contrast, this Gravi-Pati-Salam-Weak has two Weak charges and one Gravitational charge, and is able to generate another 3-simplex (u_L, d_L, u_R, d_R) (see Table 8). The 4-simplex of Gravi-Hyperflavor-Weak in Table 8 is partially inspired by Hyperflavor-Weak Theory [4], and the fact that the larger quasi-exceptional algebras, *E10* and *E12*, require the five-fold symmetry of a pentachoron particle multiplet. This secondary conserved quantum number is important, and has the interpretation of a right-handed weak isospin projection operator, $T_{3R} = (\sqrt{3} T_{3HF} + \sqrt{6} T_{8HF})/3$. The fifth vertex of this 4-simplex has unusual characteristics.

Table 8 – A 4-Simplex (for *E10* and *E12*) or 3-Simplex (for *E8*) of Gravi-Weak

Dimensions	t	ba	bb	bc			t	ba	bc
Bosons	Z^0	z_1^0	z_2^0	G			Z^0	B_R^0	G
Charges → ↓ Fermions	T_{3L}	$\sqrt{3} T_{3HF}$	$\sqrt{6} T_{8HF}$	$\sqrt{10} T_G$	T_{3R}	T'_G	T_{3L}	$\sqrt{3} T_{3PS}$	$\sqrt{6} T_G$
$f_{1L}^{\wedge} = (u_L, \bar{e}_L)$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$
$f_{1L}^{\vee} = (d_L, \bar{\nu}_{eL})$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$
$f_{1R}^{\wedge} = (u_R, \bar{e}_R)$	0	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	1	$\frac{1}{2}$
$f_{1R}^{\vee} = (d_R, \bar{\nu}_{eR})$	0	0	$-\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$-\frac{3}{2}$
$sf_1 = (sq, s\bar{l})$	0	0	0	-2	0	$-\frac{1}{2}$	undefined		
$\bar{f}_{1L}^{\wedge} = (e_L, \bar{u}_L)$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
$\bar{f}_{1L}^{\vee} = (\nu_{eL}, \bar{d}_L)$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
$\bar{f}_{1R}^{\wedge} = (e_R, \bar{u}_R)$	0	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	-1	$-\frac{1}{2}$
$\bar{f}_{1R}^{\vee} = (\nu_{eR}, \bar{d}_R)$	0	0	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$\frac{3}{2}$
$s\bar{f}_1 = (sl, s\bar{q})$	0	0	0	2	0	$\frac{1}{2}$	undefined		

This new "particle" appears to have properties equivalent to a "scalar fermion" [9]. It is a "fermion" because it belongs to this fundamental particle multiplet. But it has quantum numbers that imply a zero spin: $T_{3L} = 0$ and $T_{3R} = 0$, thus it has neither a left-handed nor a right-handed isospin projection. Does $s_F = T_{3L} + T_{3R} + T'_G = \pm\frac{1}{2}, \pm\frac{3}{2}, \dots$ define a generalized Gravi-Weak Fermion? Are the spin projections for these new quanta hidden in a Hyperspace dimension? Do these "scalar fermions" manifest themselves as tachyons, BRST or Faddeev-Papov ghosts, or physical particles? The designation for these new quanta in Table 8 is preceded by an "s" to indicate generation-dependent scalar fields $(sf_1, s\bar{f}_1, sf_2, s\bar{f}_2, sf_3, s\bar{f}_3)$ with all four electro-color quantum numbers (r, g, b, w) . A 4-simplex (Figure 4) plus its dual contains 10 particle states $(f_L^\wedge, f_L^\vee, f_R^\wedge, f_R^\vee, sf, \bar{f}_L^\wedge, \bar{f}_L^\vee, \bar{f}_R^\wedge, \bar{f}_R^\vee, s\bar{f})$. These particles may be interconnected with the forty-five operators of $SO(10)$ or E_5 . Table 9 enumerates the Electro-Color-Gravi-Weak quantum numbers for the first generation of fermions. The upcoming Sections 4.3 and 6.3 will also account for generations, spin and anti-matter. Note the new conservation laws: $g'_8 = (\sqrt{3}g_8 - \frac{3}{2}Y')/3 = 0, \pm\frac{1}{2}, \dots$, $T_{3R} = (\sqrt{3}T_{3HF} + \sqrt{6}T_{8HF})/3 = 0, \pm\frac{1}{2}, \dots$ and $T'_G = (\sqrt{10}T_G + F_3)/5 = 0, \pm\frac{1}{2}, \dots$. In Figure 4, the secondary gravity quantum number T'_G is defined with the expectation that Gravity and a new WIMP-Gravity will collectively comprise a Clifford bivector and mix charges such that $T'_G = (\sqrt{10}T_G \mp \frac{1}{2})/5 = 0, \pm\frac{1}{2}$, etc. (see Tables 8 & 9).

Having defined a right-handed weak isospin, it is also logical to define a right-handed weak hypercharge: $Q = T_{3L} + \frac{1}{2}Y_L = T_{3R} + \frac{1}{2}Y_R$, so that $Y_R = 2Q - T_{3R}$. Now we can state the electric charges of these new scalar fermions:

$$Q = \begin{cases} \frac{1}{6} & \text{for } sf \text{ of color } = (r, g, b) \\ -\frac{1}{2} & \text{for } sf \text{ of color } = w \\ -\frac{1}{6} & \text{for } s\bar{f} \text{ of color } = (\bar{r}, \bar{g}, \bar{b}) \\ \frac{1}{2} & \text{for } s\bar{f} \text{ of color } = \bar{w} \end{cases} \quad (14)$$

Thus, these sf scalar fermions are not the supersymmetric partners to the known fermions.

We may now define more bosons as translation vectors in Electro-Color-Gravi-Weak quantum numbers. Table 10 defines the $SU(5)$ X and Y Leptoquark bosons in color-hypercharge-weak isospin quantum numbers. Table 11 defines the Hyperflavor w bosons and the $SU(7)$ Gravi-Weak "V bosons" in weak isospin-hyperflavor-gravity quantum numbers. New Feynman

diagrams reviewing these transitions are shown in Section 5.3. Table 12 is a review of various transitions in which these “bosons” are involved. To read this table, find the $X^{\frac{5}{3},c\bar{w}}$ that is located at the intersection of the u_L^c row and the e_L^w column. The interpretation is that an $X^{\frac{5}{3},c\bar{w}}$ of (electric and color) charge of ($\frac{5}{3}$ and “color-anti-white”) operating on an e_L^w of color “white” yields a u_L^c of generic color “color”, or $u_L^c = X^{\frac{5}{3},c\bar{w}} e_L^w$, where past (future) is on the right (left).

Table 9 – Electro-Color-Gravi-Weak Quantum Numbers for Select Fundamental Fermions

Charges → ↓ Fermions	g_3	$\sqrt{3} \times$ g_8	$-\frac{3}{2} \times$ Y'	T_{3L}	$\sqrt{3} \times$ T_{3HF}	$\sqrt{6} \times$ T_{8HF}	$\sqrt{10} \times$ T_G	F_3	g'_8	T_{3R}	T'_G
u_L^r	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0
u_L^g	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0
u_L^b	0	-1	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0
e_L^w	0	0	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0
d_L^r	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0
d_L^g	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0
d_L^b	0	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0
ν_{eL}^w	0	0	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0
u_R^r	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	1	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	0
u_R^g	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	1	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	0
u_R^b	0	-1	$-\frac{1}{2}$	0	1	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0
e_R^w	0	0	$\frac{3}{2}$	0	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0
d_R^r	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$-\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0
d_R^g	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$-\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0
d_R^b	0	-1	$-\frac{1}{2}$	0	0	$-\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0
ν_{eR}^w	0	0	$\frac{3}{2}$	0	0	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0
sq_1^r	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	-2	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$
sq_1^g	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	-2	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$
sq_1^b	0	-1	$-\frac{1}{2}$	0	0	0	-2	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$
sl_1^w	0	0	$\frac{3}{2}$	0	0	0	2	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$

Does $s_B = \Delta T_{3L} + \Delta T_{3R} + \Delta T'_G = 0, \pm 1, \dots$ define a generalized Gravi-Weak Boson in the reciprocal E_8 lattice? These new V "bosons" have properties similar to fermions (Table 11) and electric charges that are not expected by Minimal Supersymmetry, that are given by $Q = \Delta Y'/2 + \Delta T_{3L} + (\sqrt{3} \Delta T_{3HF} + \sqrt{6} \Delta T_{8HF})/3$, thus correcting the relationship between electric charge and hypercharge in the context of Hyperflavor-Electroweak (HEW) unification.

Figure 4 – A Distorted Petrie Polygon of a 4-Simplex of Gravi-Hyperflavor-Weak

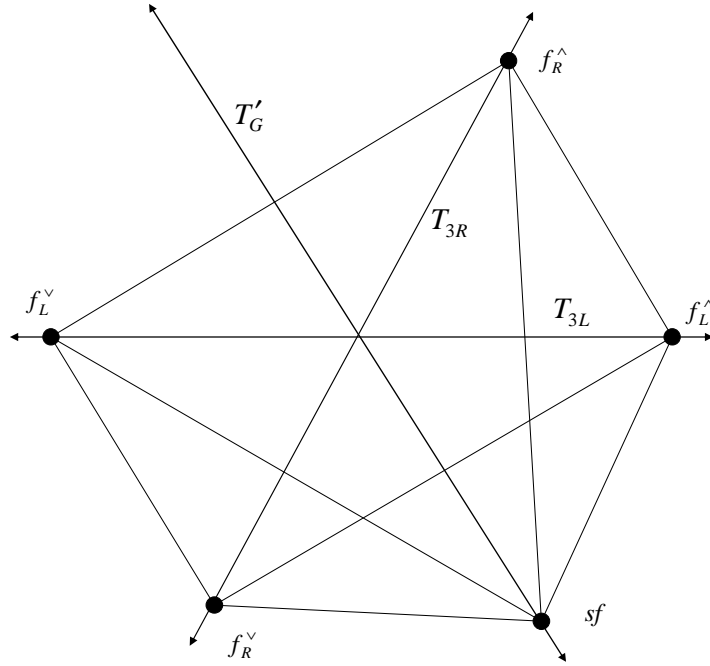


Table 10 – The Leptoquark Bosons of $SU(5)$ as Translation Vectors

Charges \rightarrow \downarrow Bosons	Δg_3	$\sqrt{3} \times$ Δg_8	$-\frac{3}{2} \times$ $\Delta Y'$	ΔT_{3L}	Charges \rightarrow \downarrow Bosons	Δg_3	$\sqrt{3} \times$ Δg_8	$-\frac{3}{2} \times$ $\Delta Y'$	ΔT_{3L}
$X^{\frac{5}{3}, r\bar{w}}$	$\frac{1}{2}$	$\frac{1}{2}$	-2	1	$X^{-\frac{5}{3}, w\bar{r}}$	$-\frac{1}{2}$	$-\frac{1}{2}$	2	-1
$X^{\frac{5}{3}, g\bar{w}}$	$-\frac{1}{2}$	$\frac{1}{2}$	-2	1	$X^{-\frac{5}{3}, w\bar{g}}$	$\frac{1}{2}$	$-\frac{1}{2}$	2	-1
$X^{\frac{5}{3}, b\bar{w}}$	0	-1	-2	1	$X^{-\frac{5}{3}, w\bar{b}}$	0	1	2	-1
$Y^{-\frac{1}{3}, r\bar{w}}$	$\frac{1}{2}$	$\frac{1}{2}$	-2	-1	$Y^{\frac{1}{3}, w\bar{r}}$	$-\frac{1}{2}$	$-\frac{1}{2}$	2	1
$Y^{-\frac{1}{3}, g\bar{w}}$	$-\frac{1}{2}$	$\frac{1}{2}$	-2	-1	$Y^{\frac{1}{3}, w\bar{g}}$	$\frac{1}{2}$	$-\frac{1}{2}$	2	1
$Y^{-\frac{1}{3}, b\bar{w}}$	0	-1	-2	-1	$Y^{\frac{1}{3}, w\bar{b}}$	0	1	2	1

Table 11 – The “Bosons” of Gravi-Hyperflavor-Weak as Translation Vectors

Charges → ↓ Bosons	ΔT_{3L}	$\sqrt{3} \times$ ΔT_{3HF}	$\sqrt{6} \times$ ΔT_{8HF}	$\sqrt{10} \times$ ΔT_G	Charges → ↓ Bosons	ΔT_{3L}	$\sqrt{3} \times$ ΔT_{3HF}	$\sqrt{6} \times$ ΔT_{8HF}	$\sqrt{10} \times$ ΔT_G
W^-, w_1^-	-1	0	0	0	W^+, w_1^+	1	0	0	0
w_2^-	0	-1	-2	0	w_2^+	0	1	2	0
$w_1'^-$	$-\frac{1}{2}$	$-\frac{3}{2}$	0	0	$w_1'^+$	$\frac{1}{2}$	$\frac{3}{2}$	0	0
$w_2'^-$	$-\frac{1}{2}$	$\frac{1}{2}$	-2	0	$w_2'^+$	$\frac{1}{2}$	$-\frac{1}{2}$	2	0
w^0	$-\frac{1}{2}$	$\frac{3}{2}$	0	0	w^{0*}	$\frac{1}{2}$	$-\frac{3}{2}$	0	0
w'^0	$-\frac{1}{2}$	$-\frac{1}{2}$	2	0	w'^{0*}	$\frac{1}{2}$	$\frac{1}{2}$	-2	0
$V_L^{\wedge, -\frac{1}{2}}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{5}{2}$	$V_L^{\wedge, \frac{1}{2}}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{2}$
$V_L^{\vee, -\frac{1}{2}}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{2}$	$V_L^{\vee, \frac{1}{2}}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{5}{2}$
$V_R^{\wedge, -\frac{1}{2}}$	0	-1	$-\frac{1}{2}$	$-\frac{5}{2}$	$V_R^{\wedge, \frac{1}{2}}$	0	1	$\frac{1}{2}$	$\frac{5}{2}$
$V_R^{\vee, -\frac{1}{2}}$	0	0	$-\frac{3}{2}$	$\frac{5}{2}$	$V_R^{\vee, \frac{1}{2}}$	0	0	$\frac{3}{2}$	$-\frac{5}{2}$

Table 12 – Transitions in which Gravi-Hyperflavor-Electroweak “Bosons” Participate

Initial → ↓ Final	u_L^c	e_L^w	d_L^c	ν_{eL}^w	u_R^c	e_R^w	d_R^c	ν_{eR}^w	sq_1^c	sl_1^w
u_L^c	I	$X^{\frac{5}{3}, c\bar{w}}$	W^+, w_1^+	?	w^0	?	w_2^+	?	$V_L^{\wedge, \frac{1}{2}}$?
e_L^w	$X^{-\frac{5}{3}, w\bar{c}}$	I	?	W^-, w_1^-	?	w^0	?	w_2^-	?	$V_L^{\wedge, -\frac{1}{2}}$
d_L^c	W^-, w_1^-	?	I	$Y^{-\frac{1}{3}, c\bar{w}}$	$w_1'^-$?	w'^0	?	$V_L^{\vee, -\frac{1}{2}}$?
ν_{eL}^w	?	W^+, w_1^+	$Y^{\frac{1}{3}, w\bar{c}}$	I	?	$w_1'^+$?	w'^0	?	$V_L^{\vee, \frac{1}{2}}$
u_R^c	w^0	?	$w_1'^+$?	I	$X^{\frac{5}{3}, c\bar{w}}$	w_2^+	?	$V_R^{\wedge, \frac{1}{2}}$?
e_R^w	?	w^0	?	$w_1'^-$	$X_1^{-\frac{5}{3}, w\bar{c}}$	I	?	w_2^-	?	$V_R^{\wedge, -\frac{1}{2}}$
d_R^c	$w_2'^-$?	w'^0	?	w_2^-	?	I	$Y^{-\frac{1}{3}, c\bar{w}}$	$V_R^{\vee, -\frac{1}{2}}$?
ν_{eR}^w	?	$w_2'^+$?	w'^0	?	w_2^+	$Y^{\frac{1}{3}, w\bar{c}}$	I	?	$V_R^{\vee, \frac{1}{2}}$
sq_1^c	$V_R^{\wedge, -\frac{1}{2}}$?	$V_R^{\vee, \frac{1}{2}}$?	$V_L^{\wedge, -\frac{1}{2}}$?	$V_L^{\vee, \frac{1}{2}}$?	I	?
sl_1^w	?	$V_R^{\wedge, \frac{1}{2}}$?	$V_R^{\vee, -\frac{1}{2}}$?	$V_L^{\wedge, \frac{1}{2}}$?	$V_L^{\vee, -\frac{1}{2}}$?	I

4.3 Generations and Generatons

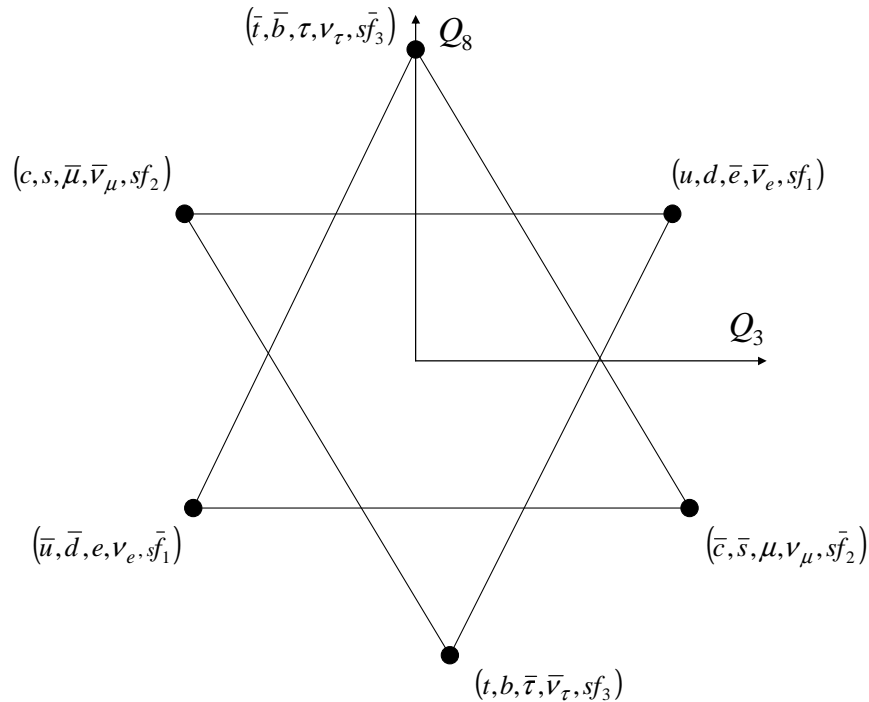
Note that these dual 2-simplices (nested equilateral triangles) of Generations (Table 13 and Figure 5) duplicate Lisi's "trality". Lisi's w quantum number has a similar effect as this Q_8 "generaton", and only gives the triality result because he used nested simplices to yield an effect similar to the Q_3 generaton. Lisi's w might be sufficient to yield the Cabibbo angle, but not the entire CKM quark and/ or PMNS neutrino mixing matrix.

Table 13 – Dual 2-Simplices of Generations

Dimensions	ad	ae
Bosons	Q_3	Q_8
Charges \rightarrow \downarrow Gen's	Q_3	$\sqrt{3}Q_8$
1 st Gen	$\frac{1}{2}$	$\frac{1}{2}$
2 nd Gen	$-\frac{1}{2}$	$\frac{1}{2}$
3 rd Gen	0	-1

Dimensions	ad	ae
Bosons	Q_3	Q_8
Charges \rightarrow \downarrow Gen's	Q_3	$\sqrt{3}Q_8$
Anti-1 st Gen	$-\frac{1}{2}$	$-\frac{1}{2}$
Anti-2 nd Gen	$\frac{1}{2}$	$-\frac{1}{2}$
Anti-3 rd Gen	0	1

Figure 5 – Dual 2-Simplices of Generations



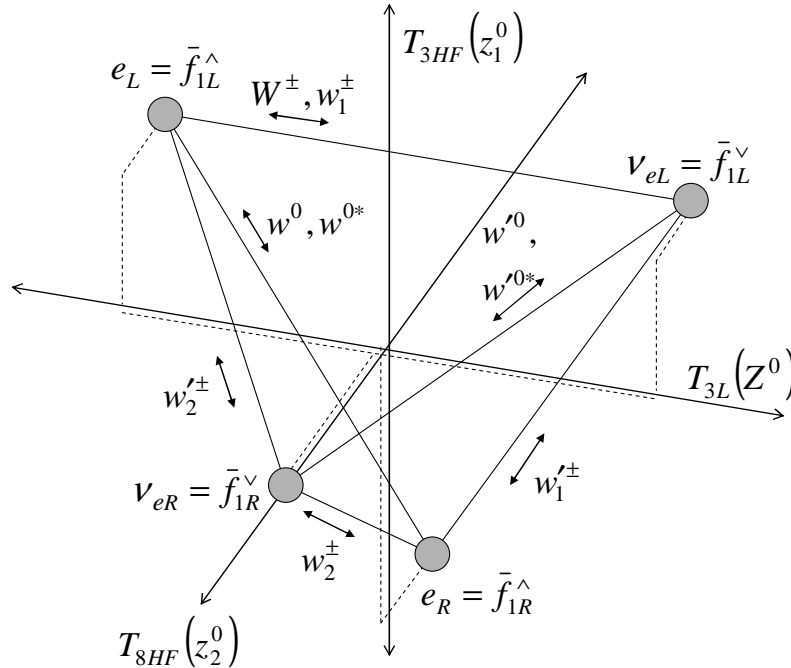
5. New Theories

5.1 Hyperflavor – Weak Theory

Figure 6 demonstrates this new Hyperflavor $SO(2,4)_{HF}$ Weak Theory with fifteen components that decompose into three charge dimensions (C_1, C_2, C_3) (from Equation 4) that mix with ϕ to form (Z^0, z_1^0, z_2^0, H) and two 6-plets, each with three alternate basis vectors and opposites: $(w^0, w_1^-, w_2^-, w^{0*}, w_1^+, w_2^+)$ and $(w'^0, w_1'^-, w_2'^-, w'^{0*}, w_1'^+, w_2'^+)$. This $SO(2,4)_{HF}$ is the Anti-de Sitter (AdS) group.

We assume that the rank-4 Georgi-Glashow $SU(5)_{GUT}$ exists in four-dimensional Spacetime and the proposed rank-6 $SU(7)_{GUT}$ of Postulate 3.2 (also see Equation 4) exists in a six-dimensional space. The difference between this six-dimensional Boson GUT space and four-dimensional Spacetime yields a two-dimensional M2-brane of Weak-brane that allows anyonic spin statistics [10]. Thus, we understand the confusion of why the “scalar fermions” (sq, sl) of Table 9 look like bosonic tachyons, and why the Gravi-Weak “ V bosons” (V_L, V_R) of Table 11 look like fermions. We propose that this M2-brane of Weak-brane contains the AdS group [11].

Figure 6 – A Tetrahedron of Leptons and Hyperflavor-Weak Bosons



5.2 The Unified CKM-PMNS-Munroe Mixing Matrix

$$\begin{pmatrix} d'^{-\frac{1}{3},c} \\ s'^{-\frac{1}{3},c} \\ b'^{-\frac{1}{3},c} \\ \nu_e^{0,w} \\ \nu_\mu^{0,w} \\ \nu_\tau^{0,w} \\ sd'^{\frac{1}{6},c} \\ s\nu'^{-\frac{1}{2},w} \end{pmatrix} = \begin{pmatrix} D_1^0 & T_1^{0*} & T_2^{0*} & U_1^{-\frac{1}{3},c\bar{w}} & U_4^{-\frac{1}{3},c\bar{w}} & U_7^{-\frac{1}{3},c\bar{w}} & R_1^{-\frac{1}{2}} & S_1^{\frac{1}{6},c\bar{w}} \\ T_1^0 & D_2^0 & T_3^{0*} & U_2^{-\frac{1}{3},c\bar{w}} & U_5^{-\frac{1}{3},c\bar{w}} & U_8^{-\frac{1}{3},c\bar{w}} & R_2^{-\frac{1}{2}} & S_2^{\frac{1}{6},c\bar{w}} \\ T_2^0 & T_3^0 & D_3^0 & U_3^{-\frac{1}{3},c\bar{w}} & U_6^{-\frac{1}{3},c\bar{w}} & U_9^{-\frac{1}{3},c\bar{w}} & R_3^{-\frac{1}{2}} & S_3^{\frac{1}{6},c\bar{w}} \\ U_1^{\frac{1}{3},w\bar{c}} & U_2^{\frac{1}{3},w\bar{c}} & U_3^{\frac{1}{3},w\bar{c}} & D_4^0 & T_4^{0*} & T_5^{0*} & S_4^{-\frac{1}{6},w\bar{c}} & R_4^{\frac{1}{2}} \\ U_4^{\frac{1}{3},w\bar{c}} & U_5^{\frac{1}{3},w\bar{c}} & U_6^{\frac{1}{3},w\bar{c}} & T_4^0 & D_5^0 & T_6^{0*} & S_5^{-\frac{1}{6},w\bar{c}} & R_5^{\frac{1}{2}} \\ U_7^{\frac{1}{3},w\bar{c}} & U_8^{\frac{1}{3},w\bar{c}} & U_9^{\frac{1}{3},w\bar{c}} & T_5^0 & T_6^0 & D_6^0 & S_6^{-\frac{1}{6},w\bar{c}} & R_6^{\frac{1}{2}} \\ R_1^{\frac{1}{2}} & R_2^{\frac{1}{2}} & R_3^{\frac{1}{2}} & S_4^{\frac{1}{6},c\bar{w}} & S_5^{\frac{1}{6},c\bar{w}} & S_6^{\frac{1}{6},c\bar{w}} & D_7^0 & U_{10}^{\frac{2}{3},c\bar{w}} \\ S_1^{-\frac{1}{6},w\bar{c}} & S_2^{-\frac{1}{6},w\bar{c}} & S_3^{-\frac{1}{6},w\bar{c}} & R_4^{-\frac{1}{2}} & R_5^{-\frac{1}{2}} & R_6^{-\frac{1}{2}} & U_{10}^{-\frac{2}{3},w\bar{c}} & D_8^0 \end{pmatrix} \begin{pmatrix} d^{-\frac{1}{3},c} \\ s^{-\frac{1}{3},c} \\ b^{-\frac{1}{3},c} \\ \nu_e^{0,w} \\ \nu_\mu^{0,w} \\ \nu_\tau^{0,w} \\ sd^{\frac{1}{6},c} \\ s\nu^{-\frac{1}{2},w} \end{pmatrix} \quad (15)$$

This CKM-PMNS-Munroe Matrix (Equation 15) falls directly out of the unused bosons in the $SU(11)$ Boson GUT of Postulate 3.3, and decomposes into three important sub-matrices: the Cabibbo–Kobayashi–Maskawa (CKM) quark mixing matrix (Equation 16), the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) neutrino mixing matrix (Equation 17), and the Munroe tachyon mixing matrix (Equation 18). These “ D ” diagonal entries are uniquely determined via the properties of Special Orthogonal matrices and may be treated as effective particles that are related to the Q Generators. Independently, these sub-matrices cannot obey Unitarity except in the extreme mass limits of $Mass(R, S, U) \gg Mass_{eff}(D, T)$, which may be likely with $Mass(R, S, U) \sim 2 \times 10^7 \text{ GeV} / c^2$ from Ref [4], and an expected $Mass_{eff}(D, T) \leq 5 \text{ GeV} / c^2$.

CKM Quark Mixing Matrix

$$\begin{pmatrix} d'^{-\frac{1}{3},c} \\ s'^{-\frac{1}{3},c} \\ b'^{-\frac{1}{3},c} \end{pmatrix} = \begin{pmatrix} D_1^0 & T_1^{0*} & T_2^{0*} \\ T_1^0 & D_2^0 & T_3^{0*} \\ T_2^0 & T_3^0 & D_3^0 \end{pmatrix} \begin{pmatrix} d^{-\frac{1}{3},c} \\ s^{-\frac{1}{3},c} \\ b^{-\frac{1}{3},c} \end{pmatrix} \Leftrightarrow \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad (16)$$

PMNS Neutrino Mixing Matrix

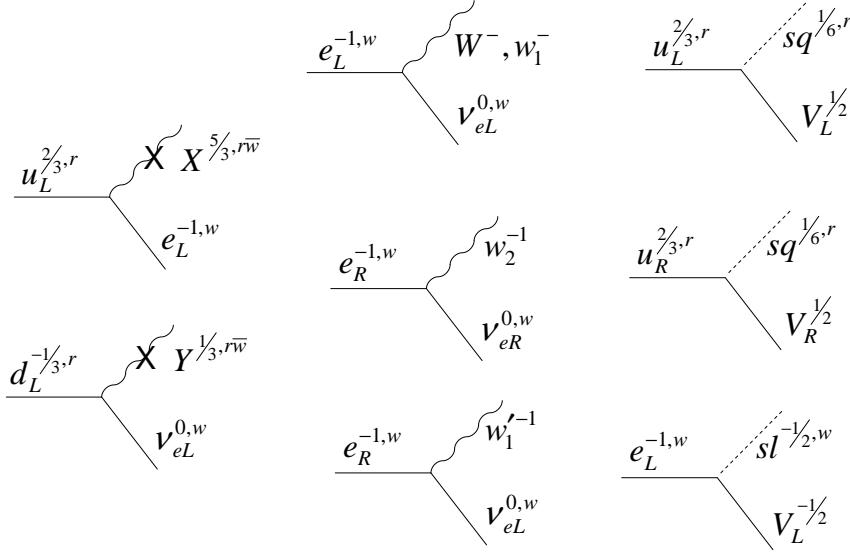
$$\begin{pmatrix} \nu_e^{0,w} \\ \nu_\mu^{0,w} \\ \nu_\tau^{0,w} \end{pmatrix} = \begin{pmatrix} D_4^0 & T_4^{0*} & T_5^{0*} \\ T_4^0 & D_5^0 & T_6^{0*} \\ T_5^0 & T_6^0 & D_6^0 \end{pmatrix} \begin{pmatrix} \nu_1^{0,w} \\ \nu_2^{0,w} \\ \nu_3^{0,w} \end{pmatrix} \Leftrightarrow \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \quad (17)$$

Munroe Tachyon Mixing Matrix

$$\begin{pmatrix} sd'^{\frac{1}{6},c} \\ s\nu'^{-\frac{1}{2},w} \end{pmatrix} = \begin{pmatrix} D_7^0 & U_{10}^{\frac{2}{3},c\bar{w}} \\ U_{10}^{-\frac{2}{3},w\bar{c}} & D_8^0 \end{pmatrix} \begin{pmatrix} sd^{\frac{1}{6},c} \\ s\nu^{-\frac{1}{2},w} \end{pmatrix} \quad (18)$$

5.3 Feynman Diagrams for the "New" Bosons of $SU(7)$

Figure 7 – Feynman Diagrams for the "New" Bosons of $SU(7)$



Note that all of these interactions are simple three-legged fermion-boson-fermion interactions because our bosons and fermions are dual lattices to each other (Postulate 6), so that bosons may be defined as differences between fermions. The only reason to have more than three legs coming off of a vertex is if we are considering non-Abelian bosonic interactions. The left-most column of diagrams in Figure 7 demonstrates the Georgi-Glashow X and Y bosons, which are not a new concept, and only a couple of sample diagrams are shown. These bosons allow proton decay, which is disallowed to a lower limit of 6.6×10^{33} years, thus requiring the masses of these X and Y bosons to be relatively large, possibly the Planck scale or the new mass scale of $\sim 2 \times 10^4$ TeV from Reference [4]. The middle column of diagrams demonstrates sample interactions involving the new Hyperflavor bosons and their subtle helicity-changing differences with Standard Electroweak Theory.

The right-most column of diagrams demonstrates sample interactions involving our new tachyonic "scalar fermions" (sq, sl) and "fermion-like bosons" (V_L, V_R). These new tachyons must be produced in pairs. We could imagine an interaction that begins like the top right diagram whereby $u_L^{2/3,r} \rightarrow sq^{1/6,r} + V_L^{1/2} \rightarrow sq^{1/6,r} + sq^{-1/6,\bar{g}} + u_L^{2/3,g}$ and the V "boson" is an unstable propagator that enforces tachyonic pair production.

6. “Theories of Everything”

6.1 A Modified $E8$ TOE

This $E8$ is different from Lisi’s in at least two important respects: 1) this fermion multiplet does not contain spinors or anti-matter and, therefore, cannot properly represent real particles, and 2) this TOE includes an $SU(9)$ “Boson GUT” that decomposes into Hyperflavor, a basic WIMP-Gravity, and a Georgi-Glashow $SU(5)$. We could reproduce spinors and fermion anti-matter with a seemingly inefficient $E8 \times F4 \times F4 \times F4$ product of Lie Algebras.

$$\begin{aligned}
 E8_{R+B} &\rightarrow SU(9) \times SO(16) \times F4_R \\
 SU(9)_{GUT} &\rightarrow SU(7)_{GUT} \times U(1)_G \times SU(2)_{WG} \text{ plus a 12-plet of } T \text{ bosons plus } U \text{ bosons} \\
 &\rightarrow SU(5)_{GUT} \times SO(2,4)_{HF} \text{ plus an 8-plet of } V \text{ bosons.} \\
 &\rightarrow SU(3)_C \times U(1)_Y \times SU(2)_L \text{ plus a 6-plet each of } X \text{ and } Y \text{ bosons.} \\
 F4_R &= (r, g, b, w)_{EC} \times (f_L^\wedge, f_L^\vee, f_R^\wedge, f_R^\vee)_{GW} \times (1st, 2nd, 3rd)_{Gen} \\
 &\quad \underbrace{(g_3, g_8, \gamma)}_{4\text{-plet} \times} \quad \underbrace{(Z, B_R, G)}_{4\text{-plet} \times} \quad \underbrace{(Q_3, Q_8) = 8 \text{ Basis}}_{3\text{-plet} = 48} \\
 &\quad (aa, ab, ac) \quad (t, ba, bb) \quad (ad, ae) = 8 \text{ Dim}
 \end{aligned} \tag{19}$$

6.2 An $E10$ TOE

This $E10$ theory is the first TOE candidate that includes two-component Weyl-van der Waerden spinors (based on Pauli σ matrices) and an $SU(11)$ “Boson GUT”. Theoretically, this TOE does not represent fermion anti-matter, or the Weak- and/ or Gravity-brane and, therefore, cannot reproduce the present value of the gravitational coupling constant.

$$\begin{aligned}
 E10_{R+B} &\rightarrow SU(11) \times SO(20) \times E6'_R \\
 SU(11)_{GUT} &\rightarrow SU(7)_{GUT} \times U(1)_G \times SU(4)_{WG} \text{ plus 12-plets of } R, S \text{ and } T, \text{ plus } U \text{ bosons} \\
 E6'_R &= (r, g, b, w)_{EC} \times (f_L^\wedge, f_L^\vee, f_R^\wedge, f_R^\vee, sf)_{GW} \times (\sigma_A, \pi_A)_{Pol} \times (1st, 2nd, 3rd)_{Gen} \\
 &\quad \underbrace{(g_3, g_8, \gamma)}_{4\text{-plet} \times} \quad \underbrace{(Z, z_1, z_2, G)}_{5\text{-plet} \times} \quad \underbrace{(F_3)}_{2\text{-plet} \times} \quad \underbrace{(Q_3, Q_8) = 10 \text{ Basis}}_{3\text{-plet} = 120} \\
 &\quad (aa, ab, ac) \quad (t, ba, bb, bc) \quad (ca) \quad (ad, ae) = 10 \text{ Dim}
 \end{aligned} \tag{20}$$

6.3 Spinors and an E12 TOE

This E12 TOE now includes four-component Dirac γ spinors, an $SU(13)$ "Boson GUT", a 3-brane for a Gravity-brane plus a 2-brane for a Generation-brane that may collectively comprise Witten's anticipated 5-brane [12]. This 5-brane may be the M5-brane that is dual to the M2-brane of Section 5.1. This $SU(13)$ introduces new force bosons that are expected to be responsible for generations. These four-component Dirac γ spinors may decompose into twistors or pairs of two-component Pauli σ spinors with primed and unprimed indices [13]. Please note that these simplices are multiplicative, and thus build-up higher-dimensional lattices. As such the $E8_R$ in Equation 21 contains many component symmetries including Lisi's triality (of Generations), a duality (of Spinors), a pentality (of Gravi-Weak Fermions), and an octality (of Electro-Color times Matter/ Anti-Matter) (see the Extended Dynkin Diagrams of Figure 1: $240 = 8 \times 30 = 8 \times (2 \times 3 \times 5)$).

Table 14 – A 3-Simplex of Spinors Collapsing Into Two 1-Simplices

Dimensions	ca	cb	cc	
Bosons	F_3	F_8	F_{15}	
Charges → ↓ Spinors	F_3	$\sqrt{3}F_8$	$\sqrt{6}F_{15}$	F'_8
σ_A	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0
π_A	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0
$\sigma_{A'}$	0	-1	$-\frac{1}{2}$	$-\frac{1}{2}$
$\pi_{A'}$	0	0	$\frac{3}{2}$	$\frac{1}{2}$

Dimensions	ca	
Bosons	F_3	
Charges → ↓ Spinors	F_3	F'_8
matter σ_1	$\frac{1}{2}$	0
matter σ_2	$-\frac{1}{2}$	0
anti-matter σ_1	0	$-\frac{1}{2}$
anti-matter σ_2	0	$\frac{1}{2}$

$$E12_{R+B} \rightarrow SU(13) \times SO(24) \times E8_R$$

$$SU(13)_{GUT} \rightarrow SU(11)_{GUT} \times SU(3)_P \text{ plus 20-plets of } N_{1-10} \text{ and } P_{1-10} \text{ bosons}$$

$$E8_R = (r, g, b, w)_{EC} \times (f_L^\wedge, f_L^\vee, f_R^\wedge, f_R^\vee, sf)_{GW} \times (\sigma_{A,A'}, \pi_{A,A'})_{Pol} \times (1st, 2nd, 3rd)_{Gen} \quad (21)$$

$$\begin{array}{cccc} \underbrace{4\text{-plet}} \times & \underbrace{5\text{-plet}} \times & \underbrace{(2+2)\text{-plet}} \times & \underbrace{3\text{-plet}} = 240 \\ (g_3, g_8, \gamma) & (Z, z_1, z_2, G) & (F_3, F_8, F_{15}) & (Q_3, Q_8) = 12 \text{ Basis} \\ (aa, ab, ac) & (t, ba, bb, bc) & (ca, cb, cc) & (ad, ae) = 12 \text{ Dim} \end{array}$$

This 3-brane Gravity-brane leads to 1) the definition of a 3-simplex of four-component spinors that decomposes into a 1-simplex (AB) of two-component spinors (σ_A, π_A) equivalent to (σ_1, σ_2) along the $F_3(C_3)$ axis in Figure 3 and Table 14, and a second 1-simplex (CD) of two-component $(\sigma_{A'}, \pi_{A'})$ spinors (which is equivalent to a 1-simplex of Matter/ Anti-Matter) along the collapsed $F'_8(C'_8)$ axis; 2) a proper definition for Dirac’s Large Number $\sim 10^{40}$ and the present value of the Gravitational coupling constant (based on arguments in Reference [4]); and 3) a proper brane to contain and confine WIMP-Gravity charges.

Analyzing the fundamental properties of $E12$, we have the Coxeter-Dynkin diagrams of Figure 1. Although this Coxeter-Dynkin diagram demonstrates an isomorphism between this quasi-exceptional $E12$ and $E8 \times H4$, it is interesting to observe that $E12$ contains a 5-brane (the 3^5 factor in Equation 2) that $E8$ (a 3^4 factor) does not contain. However, this diagram implies that the $E8 \times H4$ product may be more fundamental than the quasi-exceptional $E12$. Certainly, Lisi has made the $E8$ symmetry group popular. The $H4 = G4$ is the 120- / 600-cell group with a diploid hexacosichoric symmetry.

This $E8 \times H4$ isomorphism of the quasi-exceptional $E12$ has a substructure similar to the Clifford divisor algebras, Octonions and Quaternions. Octonions decompose as two $SO(5)$ tensor 10-plets (one symmetric and one anti-symmetric), two vector 5-plets (one polar and one axial), and two scalars (one real scalar and one pseudoscalar). Quaternions decompose as one anti-symmetric $SO(4)$ tensor 6-plet, two vector 4-plets (one polar and one axial), and two scalars (one real scalar and one pseudoscalar). Octonions are related to an $E8$ algebra, Manogue and Dray have attempted to construct a 10-dimensional Octonion Theory [14], and Lisi has attempted to construct an $E8$ Theory of Everything [1]. Manogue and Dray embedded Octonions into a complex 2×2 matrix, thus implying 10 dimensions. In principle, their theory could be reformulated into a 12 dimensional theory by representing the complex 2×2 matrix transformation as a real 4×4 matrix (a real Quaternion) transformation.

Previously, the author [4] made the observations that 1) Quaternions can represent the decomposition of Spacetime, and 2) Octonions contain two tensor 10-plets – one of which could contain Einstein’s ten coupled tensor Field Equations of General Relativity: $R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R + \Lambda g_{\alpha\beta} = G_{\alpha\beta} + \Lambda g_{\alpha\beta} = 8\pi T_{\alpha\beta}$ [15]. If the Quaternionic $SO(4)_Q$ tensor 6-plet represents Gravity and one of the Octonionic $SO(5)_O$ tensor 10-plets represents a related force called WIMP-Gravity (Weakly Interacting Mediating Particle Gravity – these are the “Fifthons”

of Table 3), then these forces may mix quantum numbers to form Clifford bivector interactions (implied in Section 4.2) and a massless Graviton as follows:

$$SO(5)_O \times SO(4)_Q \rightarrow U(1)_G \times SO(5,1)_{F,G} \rightarrow U(1)_G + 10_{F,G} + 5_F \quad (22)$$

where $5_F = (F_1, F_2, F_3, F_8, F_{15})$ and $10_{F,G} = (F_{A-E}, F_{A-E}^*)$ – see Tables 3 and 4.

The $SO(5,1)_{F,G}$ lies on the M5-brane that is dual to the M2-brane containing $SO(2,4)_{HF}$ and thus we have the complementary symmetries $SO(5,1)_{F,G \text{ on } M5} \approx SO(6) \approx SO(2,4)_{HF \text{ on } M2}$ that seem to define how the twelve dimensional $E12$ decomposes into a five dimensional M5-brane (of WIMP-Gravity and Generatons) plus a one dimensional string (of Gravity) plus a two dimensional M2-brane (of Hyperflavor) plus four dimensions of Spacetime. This bivector union of Gravity and WIMP-Gravity tensor components also binds the 4-dimensional Quaternion to the 8-dimensional Octonion. The Quaternion subsequently experienced an inflationary period (Reference [4] presented an inflationary phase change caused by the thermodynamics of Quantum Statistical Grand Unification and the primordial masses of the (Z^0, W^\pm) bosons) that produced a 4-dimensional Spacetime, while the Octonion inflated very little thus forming the 8-dimensional Hyperspace required for a 12-dimensional $E12$ Theory of Everything. Knowledge of the Octonion and the particles and fields confined within its surface are obscured by the smallness (and equivalent large energy scale $\Delta p \geq \hbar/\Delta x$) of the Octonion.

The $(1,5,10,10,5,1)$ symmetry of the Octonion implies that this 8-dimensional Octonion decomposes into a 5-brane, a 2-brane, and a 1-dimensional string (a $U(1)_G$ of Gravity). This symmetry is closer to the $[5,3^{5,2,2,1}]$ symmetry of $E12$ than the $[3^{4,2,1}]$ symmetry of $E8$. The $(1,4,6,4,1)$ symmetry of the Quaternion combined with the $(1,3,3,1)$ symmetry of Vector Algebra likewise implies that this 4-dimensional Quaternion decomposes into a “3-brane” of Space and a “1-dimensional string” of Time. Note that the geometrical nature of these simplices allows certain dimensions to collapse. This feature is related to BRST Theory and Clifford bivectors. One of the Spinor dimensions of $E12$ collapses to yield two 1-simplices ($F'_8 = (\sqrt{3}F_8 + \sqrt{6}F_{15})/3$, see Equation 21 and Table 14). And one of the Hyperflavor-Weak dimensions may collapse to yield a Pati-Salam Weak ($T_{3R} = (\sqrt{3}T_{3HF} + \sqrt{6}T_{8HF})/3$, see Table 8). Such a broken symmetry or dimensional collapse could explain why the scalar fermions $(sf_1, \bar{sf}_1, sf_2, \bar{sf}_2, sf_3, \bar{sf}_3)$ have not been discovered. It is interesting to compare the

number of $E_{12}_R = 672$ roots with the multi-dimensional Kissing Number problem. We find that this number of roots falls between the best eleven-dimensional bounds of 582 [16] and 870-905, and far below the 12-dimensional lower-bound of 840. Thus this E_{12} TOE collapses into eleven effective dimensions – apparently consistent with M-Theory.

6.4 An $E_8 \times H_4$ “Quiver of Quaternions” TOE

A spin-off of the E_{12} TOE is an $E_8 \times H_4$ TOE. This theory contains a “quiver of quaternions” [8] with 120 H_4 vertices each connected to an Octonion, and its similarities to E_{12} were demonstrated in Figure 1. These 120 H_4 vertices may be related to the $SU(11)$ Boson GUT 120-plet of Postulate 3.3, and these Octonions may each represent the E_8 Fermion GUT of Equation 21 (also see Table 9). This $E_8 \times H_4$ TOE lacks most of the E_{12} TOE scalar content, but may contain the Standard Model Higgs via Equations 4 and 5. Curiously, E_8 and H_4 have the same symmetries based on their Dynkin diagrams with $E_{8_R} = 240 = 8 \times (2 \times 3 \times 5)$ and $H_{4_R} = 120 = 4 \times (2 \times 3 \times 5)$, so that both symmetries exhibit two-fold “duality”, three-fold “triality”, and five-fold “pentality” symmetries.

7. Discussion

Lisi’s E_8 TOE had some theoretical shortcomings, such as: 1) Lisi’s E_8 is a Gosset lattice – the Gosset lattice has many symmetries based on its Dynkin diagram with $240 = 8 \times (2 \times 3 \times 5)$, but Lisi has not yet identified the five-fold “pentality” symmetry, 2) it is inconsistent for part of this Gosset lattice to inflate into Spacetime while another part remains a small Hyperspace, therefore all dimensions must be Hyperspace dimensions and Spacetime is not fully incorporated, and 3) Lisi included the third generation of fermions in the same $SO(16)$ as bosons and separate from the first two generations of fermions. The E_8 Gosset lattice and this paper’s simplices imply that hyperspace may have structure.

Does spacetime or hyperspace have substructure? Certainly, String Theory/ M-Theory has the substructure of p-branes. And Causal Dynamical Triangulation (CDT) has the substructure of $(D-1)$ -simplices. Because this paper’s geometries are derived from simplices, they relate directly to CDT, but may also be related to nets of simplices on p-branes. Many physicists are reluctant to support a String Theory/ M-Theory that may have as many as $\sim 10^{500}$ degrees of freedom. The point is that a TOE such as E_8 , E_{10} , E_{12} or $E_8 \times H_4$ should

reduce the number of relevant parameters to dozens, while many of the $\sim 10^{500}$ degrees of freedom are “hidden variables” that are obscured by the averaged observables of quantum mechanics, and thus tie into some of the classic ideas by Einstein (hidden variables [17]) and Dirac (large numbers [18]). A best estimate on the number of degrees of freedom in a 12-dimensional *E12* “Membrane Theory” may be obtained from an expansion of the Large Number arguments in Reference [4]: $\sim 10^{533} \sim (10^{40 \times 2})^{(4 + \frac{8}{3})}$. Although this is an extremely large number, it is not infinity – an infinite number here would lead to an infinitely weak (and incorrect) Gravitational coupling constant! Of these $\sim 10^{533}$ degrees of freedom, we have $\sim 10^{240}$ space-momentum initial conditions, $\sim 10^{80}$ time-energy initial conditions, and $\sim 10^{213}$ hidden variables (some of these variables lie on the Gravity-brane and enforce Dirac’s Large Number of $\sim 10^{40}$, while many of these may be constrained or “entangled” with other variables via geometrically conserved quantum number “fossil” remnants of a collapsed *E12* lattice with Clifford bivectors and/ or BRST Theory). Note that our Universe has $\sim 10^{213}$ hidden variables and $\sim 10^{120}$ particles, which introduces enough degeneracy that entanglement is allowed. Strings plus entanglement may explain the action at-a-distance behavior of Electromagnetism and Gravitation. It may be more appropriate to think of a “particle” as an entangled collection of lattice vibrations. The small number of TOE parameters makes the Universe intelligible, while the large number of quantum-obscured and entangled hidden variables introduces enough chaos that we can never know everything about the Universe.

Another “problem” with String Theory/ M-Theory is the failure to discover supersymmetric particles at the Tevatron or the LHC (thus far). Note that the “scalar fermions” in Section 4.2 are defined in terms of Gravity quantum numbers and appear to be completely neutral to the Hyperflavor-Weak quantum numbers of Table 8 – even more neutral than the elusive right-handed neutrinos. The author is on record [4] for stating that supersymmetric particles may exist at a proposed Gravity/ WIMP-Gravity unification energy scale of $\sim 2 \times 10^4 \text{ TeV}$ rather than the generally expected Weak energy scale of $\leq \text{TeV}$. This leads to fine tuning of 1 part in $\sim 5 \times 10^{10} = (2 \times 10^4 \text{ TeV})^2 / (0.09 \text{ TeV})^2$, which is similar to some important scaling numbers: 1) the low-energy ratio of the electromagnetic to weak couplings $\sim 2 \times 10^9$, 2) the inverse of the low-energy weak coupling $\sim 3 \times 10^{11}$, and 3) the cube root of Dirac’s Large Number $\sim 3 \times 10^{13}$, which is the number of direct lattice sites per hyperspace dimension. This energy scale of $\sim 2 \times 10^4 \text{ TeV}$ can be attained by cosmic rays, not by supercolliders.

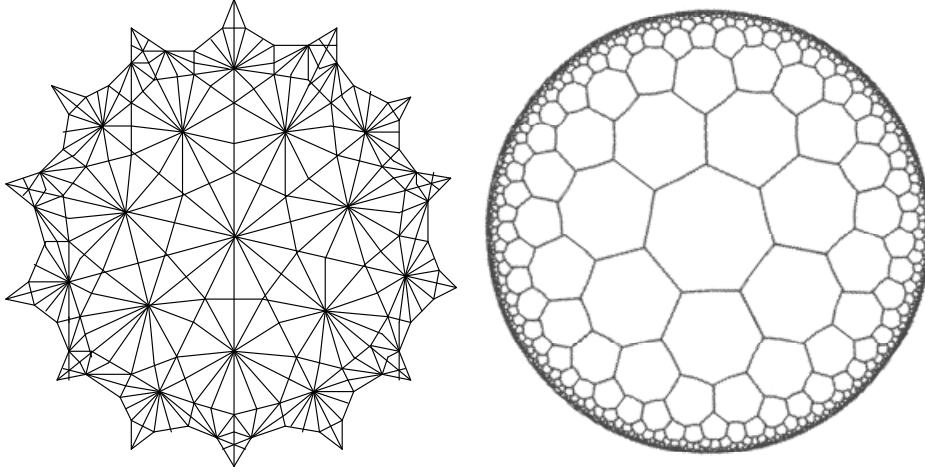
If Supersymmetry (SUSY) exists, it will require at least an $E_{12} \times E_{12}$ product of Lie algebras to contain supersymmetric particles. Postulate 6 expects supersymmetric partners to lie in dual lattices. But what is dual to our newly found E_{12} ? E_{12} has 672 roots, which implies a product of two of Klein's $\chi(7)$ curves or ten-dimensional laminated lattices Λ_{10} with an order of 336 each. The dual to Klein's $\chi(7)$ is the cubic symmetric $F_{056}B$ with an order of 56 (one-sixth of 336), but this is too far removed from the original twelve dimensional E_{12} lattice to determine a dual. E_{12} has similarities with $E_8 \times H_4$. The E_8 Gosset lattice is self-dual with 240 roots, but H_4 is represented by the 120-cell which is dual to the 600-cell. Thus, our Supersymmetric dual lattice may contain at least $840 = 240 + 600$ roots, more than our original E_{12} lattice with 672 roots.

What are our expectations for SUSY particle content? We should expect particle multiplets of scalars (intrinsic spin 0), matter fermions (intrinsic spin $\frac{1}{2}$), vector bosons (intrinsic spin 1), gravitino fermions (intrinsic spin $\frac{3}{2}$), and tensor bosons (intrinsic spin 2) – a total of five distinct particle multiplets. E_{14} has 1,008 roots, which implies a product of three of Klein's $\chi(7)$ hyperbolic curves. Thus, the product of $E_{12} \times E_{14}$ contains five $\chi(7)$ curves or Λ_{10} 's – enough to contain five distinct particle multiplets in a 26-dimensional space.

These Klein $\chi(7)$ curves have some interesting properties. If we consider $\chi(7)$ "filled heptagons" equivalent to filled atomic orbital shells, then we should expect "stability" for the following particle multiplets: $14 = 1 \times 14 \Rightarrow I_2(7)$, $112 = 8 \times 14 \Rightarrow 4 \times SO(8)$, $210 = 15 \times 14 \Rightarrow 2 \times SO(15)$, $308 = 22 \times 14$, and a filled $\chi(7) \sim 336$. We might also expect "partial stability" for the following shapes: $28 = 4 \times 7 \Rightarrow SO(8)$ star, $42 = 6 \times 7 \Rightarrow 2 \times SO(7)$ polygon, and $140 = 20 \times 7$ polygon. If we count our vector boson content, we have $(g_{1-8}, \gamma, Z, W^\pm)^{\wedge \vee}$ and $(Z, W^\pm)^L$ for a total of 27 quantum states including longitudinal dgf's derived from scalars, or 24 quantum states not including longitudinal dgf's. Either way of counting, we are short of the 28 dgf's required for partial stability, and should thus expect another vector (or scalar?) boson to exist in this low-energy realm. If we count our matter fermion content, we have $(u, d, c, s, t, b)_{L,R;r,g,b}^{\wedge \vee}$, $(\bar{u}, \bar{d}, \bar{c}, \bar{s}, \bar{t}, \bar{b})_{L,R;r,g,b}^{\wedge \vee}$, $(e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau)_{L,R}^{\wedge \vee}$, and $(\bar{e}, \bar{\mu}, \bar{\tau}, \bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau)_{L,R}^{\wedge \vee}$ for 192 quantum states, also short of the "stable" 210-plet.

Our E_{14} contains a factor of $I_2(7)$ which might be the central heptagon or "centralon" of each of these Klein $\chi(7)$ curves, and thus tie these supersymmetric particle multiplets together. Two interesting observations about this centralon include 1) $E_{12} \times E_{14}$ has

Figure 8 – Klein’s $\chi(7)$ Curve and Hyperbolic Heptagonal Tiling



Klein's $\chi(7)$

Heptagonal Tiling

1,680 = 5 × 336 (five $\chi(7)$'s) = 7 × 240 (seven E_8 's) roots so that each of the seven edges of the centralon connects with an E_8 algebra, and 2) comparison of Klein’s $\chi(7)$ curve with Heptagonal Hyperbolic Tiling (Figure 8) yields a fractal $\chi(7)$ order of $336 + 16k$ [19], where $k = 0.18034$, so that five $\chi(7)$'s have an order of $5 \times (336 + 16k) = 1,680 + 80k = 1,680 + 14.4$, and our remainder of $14.4 \approx 14$ is an $I_2(7)$ centralon in addition to the roots of $E_{12} \times E_{14}$.

The pattern of simplices that collectively comprise this E_{12} TOE imply that space is a 3-simplex of Electro-Color that is designated by dimensions $aa(1)$, $ab(2)$ and $ac(3)$, the Weak-brane is an M2-brane plus a string designated by dimensions $ba(5)$, $bb(6)$ and $bc(7)$ that couples with Time (designated by dimension $t(4)$) to form the 4-simplex of Gravi-Hyperflavor-Weak, the Gravity-brane is a 3-brane designated by dimensions $ca(8)$, $cb(9)$ and $cc(10)$ that forms the 3-simplex of Spinors, and the Generation-brane is a 2-brane designated by dimensions $ad(11)$ and $ae(12)$ that forms the 2-simplex of Generations. This numbering system for dimensions reflects the possibility that the 26 dimensions of String theory decompose first into time and five quintets (5-branes) of Space/ Strings, and then the ten most important dimensions of String theory decompose into time and three triplets (3-branes) of Space/ Strings [4]. Organizing these dimensions into branes is equivalent to building a “periodic table” of dimensions, and is expected

to look something like: $aa(1)$, $ab(2)$, $ac(3)$, $t(4)$, $ba(5)$, $bb(6)$, $bc(7)$, $ca(8)$, $cb(9)$, $cc(10)$, $ad(11)$, $ae(12)$, $bd(13)$, $be(14)$, $cd(15)$, $ce(16)$, $dd(17)$, $de(18)$, $da(19)$, $db(20)$, $dc(21)$, $ea(22)$, $eb(23)$, $ec(24)$, $ed(25)$ and $ee(26)$. Important sub-GUT's include a four-dimensional (aa , ab , ac , t) $SU(5)_{GUT}$, a seven-dimensional (add ba , bb , bc) $SU(7)_{GUT} \times U(1)_G$, a ten-dimensional (add ca , cb , cc) $SU(11)_{GUT}$, a twelve-dimensional (add ad , ae) $SU(13)_{GUT}$, a 14-dimensional (add bd , be) $SU(15)_{GUT}$, an 18-dimensional (add cd , ce , dd , de) $SU(19)_{GUT}$, and a 26-dimensional (add da , db , dc , ea , eb , ec , ed , ee) $SU(27)_{GUT}$ (see Table 15).

Lisi's $E8$ TOE would not reproduce the same Gravity-brane described here. Without properly representing the Gravity-brane, Lisi should not be able to reproduce extreme numbers such as Dirac's Large Number, Einstein's Cosmological Constant or the present value of the Gravitational Coupling.

The author previously filled in the $E12$ particle multiplet with Hyperflavor-Electroweak leptons and quarks [4]. Although these particles should exist at the Planck scale, and should be related to Kaluza-Klein leptons and quarks, they are not part of these fundamental particle multiplets. If we expand our simplices into regular lattices, then the fundamental particle

Table 15 – Periodic Table of Dimensions

aa(1) color	ab(2) color	ac(3) electro	t(4) weak	
ba(5) HF	bb(6) HF	bc(7) gravity		
ca(8) WIMP	cb(9) WIMP	cc(10) WIMP	ad(11) Gen	ae(12) Gen
			bd(13)	be(14)
			cd(15)	ce(16)
			dd(17)	de(18)
da(19)	db(20)	dc(21)		
ea(22)	eb(23)	ec(24)	ed(25)	ee(26)

multiplets expand into Kaluza-Klein particle multiplets that quickly expand into an infinite lattice, similar to the twelve dimensional Coxeter-Todd K_{12} lattice with 756 minimal roots (nearest-neighbor lattice sites) that expands into 4,032 long roots [6] (next-nearest-neighbor lattice sites) and so on to infinity.

In Chapter One, we chose not to consider an E_{14} or larger TOE. The seven-fold symmetries of these larger exceptional groups requires a 6-brane 6-simplex that requires combining branes in an unexpected manner (such as combining the M5-brane with another string, which may cause normalization problems), or deriving the 6-simplex from higher Hyperspace dimensions. Analyzing the isomorphism for E_{14} , we realize that $E_8 \times H_4 \times I_2(7) \rightarrow E_{14}$ (Equation 2) brings in a factor of an $I_2(7)$ heptagon group as well – clearly making E_{14} a less simple group than (the already non-simple, semi-simple) E_{12} .

It may be reasonable to consider an $E_{26} \rightarrow E_{12} \times E_{14}$ TOE in which the dimensions have evolved such that the E_{14} represents unimportant (non-physically relevant or undiscovered Supersymmetric) branes, and E_{12} represents the most important (most physically relevant) branes. These “Exceptional” Lie algebras may decay into smaller Lie algebras via the cascade demonstrated in Table 16.

Table 16 – 26-D String Theory Cascade

Dim	TOE's & sub-GUT's		Dim
26	$E_{26} \rightarrow$		26
14	$E_{14} \times$	E_{12}	12
14	$\rightarrow E_{10} \times SU(15) \times SO(28)$	$\rightarrow E_8 \times SU(13) \times SO(24)$	12
10	$\rightarrow E_{6'} \times SU(11) \times SO(20)$	$\rightarrow F_4 \times SU(9) \times SO(16)$	8
6	$\rightarrow G_2 \times SU(7) \times SO(12)$	$\rightarrow SU(5) \times SO(8)$	4
2	$\rightarrow SU(3) \times SO(4)$		

7. Conclusion

Theories of Everything (TOE's) based on E_8 , E_{10} , E_{12} , and $E_8 \times H_4$ “Exceptional” Lie Algebras were compared and contrasted. Although all of these ideas seem to have the proper framework for a TOE, this paper presented the twelve-dimensional (four of Spacetime plus eight of Hyperspace) E_{12} as a preferred candidate.

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