# Why Did Einstein Think The Arrow of Time Was Due To Probabilities?

In 1907 Einstein proposed a model for solids that was based on quantum harmonic oscillators (Springs). We create a solid from N oscillators and $q$ units of energy. We denote the oscillators with a vertical line "|" and the energy with a dot "•".

$$\underbrace{\bullet}_{1}\,|\,\underbrace{\bullet\;\bullet\;\bullet}_{2}\,|\,\underbrace{\bullet\;\bullet}_{3}$$

$q = 6$
$N = 3$

There are $q+N-1$ Symbols and the possible arrangements are found by choosing $q$ symbols from $q+N-1$.

$$\Omega(N,q) = \binom{q+N-1}{q} = \frac{(q+N-1)!}{q!(N-1)!}$$

The Binomial Coefficient grows Very fast for only a few Macrostates:

$$C(n,m) = \binom{n}{m} = \frac{n!}{m!(n-m)!}$$

$$\binom{10}{5} = \frac{10!}{5!\,5!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6}{5\cdot 4\cdot 3\cdot 2\cdot 1} = \frac{30240}{120} = 252$$

$$\binom{10}{9} = \frac{10!}{9!(10-9)!} = \frac{10!}{9!\,1!} = \frac{10\cdot 9!}{9!} = 10$$

$$\binom{10}{1} = \frac{10!}{1!(10-1)!} = \frac{10!}{1!\,9!} = \frac{10\cdot 9!}{9!} = 10$$

Symmetry: $\binom{n}{k} = \binom{n}{n-k}$

Now, lets increase $n + m$ by a factor of 10

$$\binom{10\cdot 10}{5\cdot 10} = \binom{100}{50} = \frac{100!}{50!\,50!} = \frac{100\cdot 99\cdots 52\cdot 51}{50!} = 1.009\times 10^{29}$$

$$\binom{10\cdot 10}{9\cdot 10} = \binom{10\cdot 10}{1\cdot 10} = \frac{100!}{90!} = 100\cdot 99\cdots 92\cdot 91 = 1.73\times 10^{13}$$

Notice that $\binom{10}{5}$ is only about 25 times larger than $\binom{10}{9}+\binom{10}{1}$ | $\frac{252}{10} = 25.2$

However, by Multiplying $n+m$ by a factor of 10 it would Seem plausible that 25.2 remains constant or becomes $10(25.2) = 252$, but no the difference is:

$$\frac{\binom{100}{50}}{\binom{100}{90}} = \frac{\binom{100}{50}}{\binom{100}{10}} = \frac{1.009\times 10^{29}}{1.73\times 10^{13}} \approx 1.0\times 10^{(29-13)} = 1.0\times 10^{16}$$

This Means $\binom{100}{50}$ is $\boxed{1.0\times 10^{16}}$ times larger than $\binom{100}{10}$

Why do you think the exclamation point is used?!

The number $10^{29}$ is a large number how can we get a feeling for just how big it is?

The observable universe is about 27 billion light years from end to end $27 \times 10^9$ light years $\approx 2.7 \times 10^{26}$ meters. If we think of millimeters or the distance from

(Obviously the PDF Zoom will distort this)

here to there $\|$

then $10^{29}$ of these would be roughly the length of the observable universe.

Now imagine there were 11 different destinations you could travel which we could write out as $n = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ or more simply as $n = 10$. These are also known as the Macrostates, and $m = n+1 = 10+1 = 11$. There is also a law that for the $m^{th}$ destination, macrostate, there must be $\binom{n}{m}$ roads or microstates which lead to that destination.

If we were to take any road at random what is the probability we reach the $m^{th}$ destination? In other words if we pick a microstate at random what is the probability it is the $m^{th}$ macrostate?

First we must know how many roads there are:

Total number of microstates $= \sum_{m=0}^{n} \binom{n}{m}$

If $n = 10$ then:

$\binom{10}{0} = \binom{10}{10} = 1 \cdot (2) \Rightarrow 2$ +

$\binom{10}{1} = \binom{10}{9} = 10 \cdot (2) \Rightarrow 20$ +

$\binom{10}{2} = \binom{10}{8} = 45 \cdot (2) \Rightarrow 90$ +

$\binom{10}{3} = \binom{10}{7} = 120 \cdot (2) \Rightarrow 240$ +

$\binom{10}{4} = \binom{10}{6} = 210 \cdot (2) \Rightarrow 420$ +

$\binom{10}{5} = \binom{10}{5} = 252 \Rightarrow 252$

Multiply by 2 because of Symmetry $\binom{n}{k} = \binom{n}{n-k}$

(The universe is high on entropy)

$1024$ = Total number of Microstates

So there are 1024 roads or microstates.

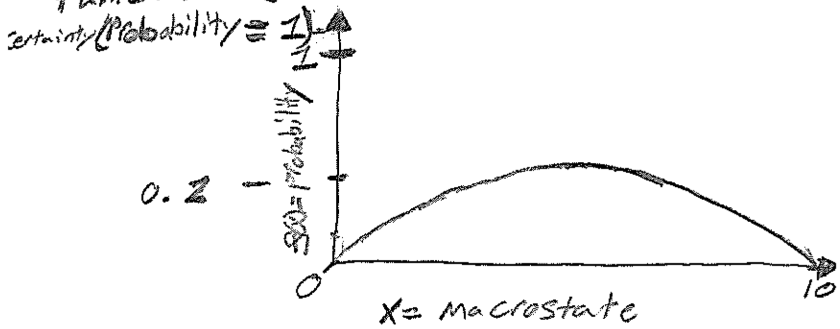
To find the probability of arriving at the $m^{th}$ destination macrostate we divide its multiplicity or number of roads going to it $\binom{n}{m}$ by the total number of all the road microstates:

$$P(m=0) = \frac{\binom{10}{0}}{1024} = \frac{1}{1024} \approx 0.001 \times 100 = 0.1\%$$

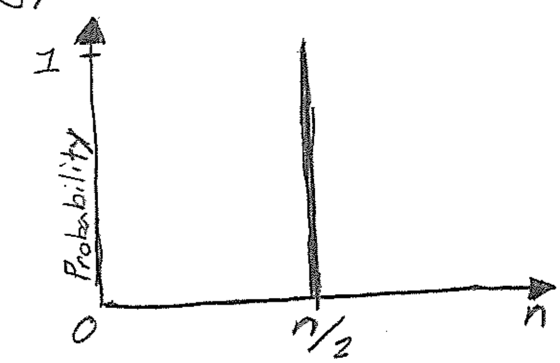
$$P(m=5) = \frac{\binom{10}{5}}{1024} = 0.24 \times 100 = 24\%$$

If $P(M=5) \approx 24\%$ and $P(M=4) = P(M=6) = \frac{210}{1024} \cdot 100 \approx 20.5\%$ then a randomly selected road leads to the $4^{th}$, $5^{th}$ or $6^{th}$ destination macrostate $20.5\% + 20.5\% + 24\% = 65\%$ of the time.

If we were to graph this for small numbers it looks like a gaussian function $f(x) = Ae^{-x^2}$




For larger values such as $\binom{100}{50}$ we saw certain macrostates become overwhelmingly probable the graphical representation becomes:




Notice how a gaussian probability function has collapsed into a definite and overwhelmingly probable macrostate. How is this explained in other branches of physics?

If our initial $n = 10$ ($M = 11$ destination macrostates) were analogous to a city then $n = 100$ ($M = 101$ destination macrostates) would result if 10 cities were combined into 1.

Previously the probability of obtaining the road (microstate) for the $M=0$ destination was small but not impossible for the $n=10$ city. However, for the $n=100$ city the multiplicity (number of roads to a destination) for $M=0$ is:

$$\binom{100}{0} = \frac{100!}{0!(100-0)!} = \frac{100!}{100!} = 1$$ (note $0! = 1$). Recall the multiplicity for the most likely macrostate ($M=50$) was an order of magnitude of $10^{29}$. This means when we compare the probability of the least likely macrostate $M=0$ with the most likely macrostate ($M=50$) for the $n=100$ city, it would be analogous to comparing 1 millimeter to the size of the observable universe! ($1:10^{29}$)