

By combining 10 cities something which previously was possible such as $M=0$ became impossible. Combining the cities is known as an irreversible process because in order to separate them we increase the multiplicity of macrostates even more than we decreased the multiplicity elsewhere. This becomes apparent when studying the Carnot Cycle or in other words any machine will not be able to have 100% perfect efficiency.

To get a better understanding of the Einstein Solid it is important to realize that just 1 gram of a pure substance contains 6.022×10^{23} atoms this would be equivalent to N . The number of quanta in the high temperature limit q is much greater than N , $q \gg N$.

If system 1 is a hot Einstein solid and system 2 is a cold solid and they are placed next to each other then the combined energy ($q_1 + q_2 = q_{total}$) has the greatest multiplicity for half q_{total} in each system. This means the two objects over time will reach the same temperature.

Since system 1 and system 2 are independent systems with multiplicities $\Omega(M)_1$ and $\Omega(M)_2$ respectively, then the combined multiplicity is:

$$\Omega(M)_1 \Omega(M)_2 = \Omega_{total}$$

For simplicity I'm going to generalize $\binom{q+N-1}{q}$ as $\binom{2q}{q}$ (This will not give the correct result because I'm neglecting the number of oscillators N in each system.)

$$q_1 = n \quad \& \quad q_2 = n \quad | \quad q_1 + q_2 = q_{total} = 2n$$

$$\Omega_{total} = \sum_{m=0}^n \Omega(n, m) = \sum_{m=0}^n \binom{n}{m} \quad \& \quad \Omega_{total} = \sum_{m=0}^n \Omega(n, m) = \sum_{m=0}^n \binom{n}{m}$$

Remember the systems are coupled so if system 1 has $2n$ energy then system 2 has $2n - 2n = 0$ energy.

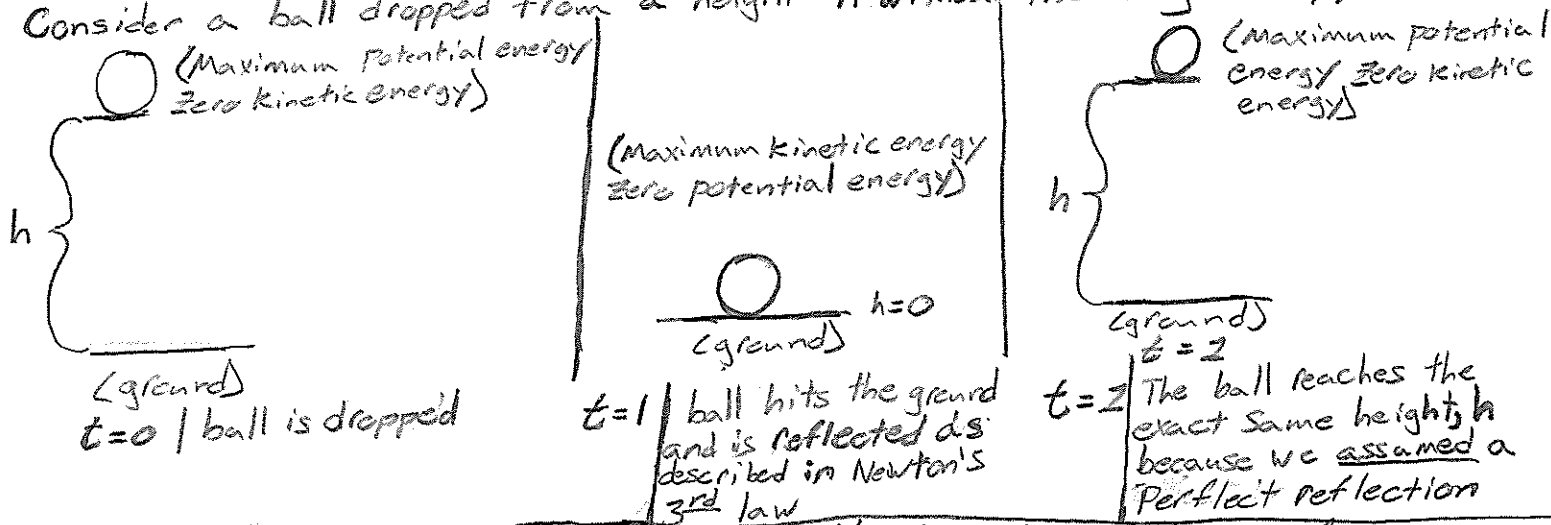
Energy in System 1 q_1	Multiplicity of System 1 $\Omega(1)$	Energy in System 2 q_2	Multiplicity of System 2 $\Omega(2)$	Multiplicity of the Combined Systems $\Omega_{total} (\Omega_1 \Omega_2 = \Omega_{tot})$
0	1	2n	1	$\Omega(1, 2) = 1 \cdot n$
1	n	2n-1	n	$\Omega(1, 2) = n \cdot n$
⋮	⋮	⋮	⋮	⋮
2n-1	$\binom{2n}{n}$	n	$\binom{2n}{n}$	$\Omega(1+2) = \binom{2n}{n} \binom{2n}{n}$
⋮	⋮	⋮	⋮	⋮
2n-1	n	1	n	$\Omega(1+2) = n \cdot n$
2n	1	0	1	$\Omega(1+2) = 1$

It becomes apparent that once a system reaches equilibrium (the macrostate with the greatest multiplicity) then it is overwhelmingly probable it will remain in equilibrium.

The Conclusion is the laws of physics are not reversible. It makes sense that most believe they are reversible, the famous laws do not include entropy for instance:

$$\vec{F} = m\vec{a}, \quad \vec{P} = m\vec{v}, \quad E = \frac{1}{2}m\vec{v} \cdot \vec{v} + C(h)$$

Consider a ball dropped from a height h without including entropy:



If $t=0$ is our initial condition and if there are perfect reflections then our initial condition is identical to our final condition; $t(0) = t(2)$, $t_i = t_f$. When we include entropy our initial condition (the ball at height h with maximum potential energy and zero kinetic energy) is not identical to our final condition (ball is on the ground at rest with zero kinetic and potential energy) $t_i \neq t_f$

The reason the ball comes to rest is attributed to entropy. When the ball bounces from striking the ground some of its kinetic energy (ΔE) undergoes a large increase in entropy and increases the temperature of the surroundings (sound too). The total energy is conserved so if it loses energy on each bounce it will eventually lose all the potential/kinetic energy and come to rest. $h \rightarrow 0$

The arrow of time is a result of irreversible changes which create distinct and separate initial and final conditions.

I am not certain that Einstein thought the same; but considering that he published the Einstein solid two years before his agreement to disagree with Ritz, then he must of thought of something similar.

The logarithm is necessary for manipulating large and very large numbers. When using large and very large numbers we safely assume numbers that are not exponents are zero.

Large Numbers: 10^n

Notice $10^n + 5 \approx 10^n$ (if $n=1$ it is false but it holds very well for larger n 's)

Very Large Numbers: 10^{10^n} (Exponent's of Exponents)

Obviously even for $n=1$ $10^{10^n} + 5 \approx 10^{10^n}$

If $n > 1$ then we safely assume $10^{10^n} + 10^m \approx 10^{10^n}$

In fact multiplying a very large number by a large number essentially leaves the very large number unchanged:

$$10^{10^n} \cdot 10^m = 10^{(10^n + m)} \approx 10^{10^n}$$

How do we effectively handle large and very large numbers?

By taking the logarithm of large and very large numbers (like the numbers that result from the binomial coefficient) we can easily handle them:

$$\log(10^n) = n \log(10) \quad \log(10^{10^n}) = 10^n \cdot \log(10) = 10^n$$

So if the multiplicity went from 10^2 such as in $\binom{10}{5}$ to 10^{29} from $\binom{100}{50}$ we could write this compactly as $2 \xrightarrow[\text{arrow of time}]{\text{Increased}}$ 29.

Entropy is: $S = k \log \Omega$ (k is Boltzmann's constant)

So the entropy of Ω_1 is $S_1 = k \log \Omega_1$ and for Ω_2 the entropy is $S_2 = k \log \Omega_2$. Remember the combined multiplicity of Ω_1 & Ω_2 is

$\Omega_1 \cdot \Omega_2 = \Omega_{\text{total}}$. So the combined entropy is:

$$S_{1,2} = k \log(\Omega_1 \cdot \Omega_2) = k [\log(\Omega_1) + \log(\Omega_2)]$$

Remember the identity $\log(a \cdot b) = \log(a) + \log(b)$

This can be simply stated as the entropy of individual systems can be added together to give the total entropy of the combined

System: $S_{\text{total}} = S_1 + S_2 + \dots + S_{n-1} + S_n = \sum_{i=1}^n S_i$