

Indeterminacy of holographic quantum geometry

Craig J. Hogan

University of Chicago and Fermilab

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An effective theory based on wave optics is used to describe indeterminacy of position in holographic spacetime with a UV cutoff at the Planck scale. Wave functions describing spacetime positions are modeled as complex disturbances of quasimonochromatic radiation. It is shown that the product of standard deviations of two position wave functions in the plane of a holographic light sheet is equal to the product of their normal separation and the Planck length. For macroscopically separated positions the transverse uncertainty is much larger than the Planck length, and is predicted to be observable as a “holographic noise” in relative position with a distinctive shear spatial character, and an absolutely normalized frequency spectrum with no parameters once the fundamental wavelength is fixed from the theory of gravitational thermodynamics. The spectrum of holographic noise is estimated for the GEO600 interferometric gravitational-wave detector and is shown to approximately account for currently unexplained noise between about 300 and 1400 Hz. In a holographic world, this result directly and precisely measures the fundamental minimum interval of time.

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I. INTRODUCTION

Theory suggests that spacetime is holographic—that quantum geometry has a native formulation on light sheets, or two-dimensional geometrical wave fronts, with a UV cutoff or minimum length at the Planck scale [1–8]. This paper proposes a quantitative formalism to calculate the properties of a new observable phenomenon of quantum holographic geometry, “holographic noise.” An effective theory of quantum geometry adapts the theory of correlations in wave optics [9] to calculate correlations of position wave functions, the resulting indeterminacy in transverse position, and the spectrum of the noise. The method used here improves earlier derivations [10–15]: it manifestly respects Lorentz symmetry, and provides a controlled calculation of all steps connecting the fundamental holographic minimum time interval with the observable noise spectrum.

The spatial correlation of the noise has a distinctive transverse character connected with its holographic origin, and the absolute normalization of the noise is fixed by gravitational thermodynamics. The precisely predicted spectrum and specific transverse spatial character provide zero-parameter experimental signatures that distinguish holographic noise from a variety of other hypothetical types of metric indeterminacy and noise [16–22]. These holographic features appear to be not specific to the particular wave model used here to compute the effect, but to be general properties of uncertainty and nonlocality associated with holographic geometry with an ultraviolet cutoff at the Planck scale.

A simple model is used to describe the effect of holographic noise in a Michelson interferometer. The response of an interferometric gravitational-wave detector [23] is estimated in a form that allows for direct comparison with

the quoted levels of instrument noise in terms of equivalent gravitational-wave metric strain. The holographic noise spectrum estimated here agrees with currently unexplained noise in the GEO600 detector [24] in its most sensitive range of frequencies, between about 300 and 1400 Hz. If holographic noise is indeed the explanation, this result represents a direct measurement of the fundamental maximum frequency, or minimum interval of time.

II. WAVE THEORY OF TRANSVERSE POSITION

The standard framework and nomenclature of wave optics [9] maps directly onto wave mechanics of a holographic geometry. The system is represented as a field of complex oscillation amplitudes of quasimonochromatic “radiation” of wavelength λ that propagates in flat spacetime via the wave equation. This carrier radiation is not an ordinary physical field, but its complex amplitudes are used to describe correlations of wave functions encoding spatial positions in a holographic model of spacetime.

Geometrical wave surfaces or geometrical wave fronts correspond to families of two-dimensional light sheets, locally orthogonal to a nominal propagation direction. The direction is determined in the quantum geometry interpretation by an experimental measurement setup, such as an interferometer beam or optical surface. Position wave functions along this direction have a width given by $\approx \lambda$ and do not enter into the following discussion. A “complex disturbance” $\psi(x, y, t)$ defined on a surface S corresponds to a wave function for a transverse event position. The wave function ψ' on another surface S' is linearly propagated from that on S via the wave equation. (In the following, primed quantities all refer to events on S' .) The “intensity” $I = \langle \psi'^* \psi' \rangle$ at one point corresponds to a time-averaged squared modulus of the quantum-

mechanical amplitude to be at that position, identified with the probability density for event position. The optical terms “mutual intensity” J and “complex degree of coherence” μ correspond to correlation operators in quantum mechanics. Measurement of the position of a body corresponds to an interaction that reduces the wave amplitude ψ that represents the spacetime to an eigenstate of transverse position. Because wave optics describes the propagation of light, the spacetime transformation properties of the correlations are automatically consistent with Lorentz invariance.

Consider two points P'_1 and P'_2 on surface S' , corresponding to pointlike events. The disturbance at point P'_1 on surface S' due to a disturbance from a small surface element m on surface S is

$$\psi'_{m1}(t) = A_m(t - L_{m1})e^{-2\pi i(t-L_{m1})/\lambda}/L_{m1}, \quad (1)$$

where L_{m1} is the separation of the point P'_1 from m , λ is the wavelength, and the speed of light c is taken to be unity. Here, $|A_m|$ denotes the amplitude and $\arg A_m$ the phase of radiation from the m th element. An exactly similar expression applies for P'_2 . The contribution from the one patch m to the time-averaged wave function correlation at the two points on S' is given by

$$\begin{aligned} \langle \psi'_{m1}(t)\psi'^*_{m2}(t) \rangle &= \langle A_m(t - L_{m1})A_m^*(t - L_{m2}) \rangle \\ &\times e^{2\pi i(L_{m1}-L_{m2})/\lambda}/L_{m1}L_{m2}. \end{aligned} \quad (2)$$

Assuming $L_{m1} - L_{m2}$ is small compared with the coherence length of the carrier, we neglect the distance difference in the time-averaged argument of A to obtain

$$\langle \psi'_{m1}(t)\psi'^*_{m2}(t) \rangle = \langle A_m(t)A_m^*(t) \rangle e^{2\pi i(L_{m1}-L_{m2})/\lambda}/L_{m1}L_{m2}. \quad (3)$$

The overall correlator of the wave function (in optical nomenclature, the “mutual intensity” J'_{12}) is given by summing over the contributions to ψ' at P'_1 and P'_2 from all patches m, n on surface S . Assume an incoherent “source” on surface S and thereby neglect correlations in the source, so contributions in the sum with $m \neq n$ average to zero. (Quantum mechanically, this corresponds to a classically defined prepared state without fine scale phase correlations.) Then combining these expressions we write the sum as an integral over 2D elements ds on surface S , giving the mutual intensity

$$\begin{aligned} J'_{12} &= \langle \psi'_1(t)\psi'^*_{2}(t) \rangle = \sum_{m1,n2} \langle \psi'_{m1}(t)\psi'^*_{n2}(t) \rangle \\ &= \int_S ds I(s) e^{2\pi i(L_1-L_2)/\lambda}/L_1L_2, \end{aligned} \quad (4)$$

where $I(s) = \langle A(t)A^*(t) \rangle_s$ is used to denote the “source intensity” at a patch s in S , interpreted as a probability distribution for an initial position—a prepared position state on the surface S . Here L_1 and L_2 denote distances from the surface element ds to P'_1 and P'_2 . The normalized

quantum-mechanical correlator in this situation is the optical “complex degree of coherence,” which differs from J'_{12} by normalizing out the mean intensity: $\mu'_{12} = J'_{12}/\sqrt{I(P'_1)I(P'_2)}$. This is the appropriately normalized quantity for tracking spacetime correlations since physical results do not depend on the intensity of the nominal carrier radiation field.

Now express the distances L_1, L_2 in terms of 2D coordinates in the geometrical wave front surface. Let x, y denote positions of source patches in the surface S and (x'_1, y'_1) and (x'_2, y'_2) denote positions at the points P'_1, P'_2 in the surface S' , and let $\Delta x' = x'_1 - x'_2$ and $\Delta y' = y'_1 - y'_2$ denote position differences on S' . Points in each surface lie in the neighborhood of point O on S and its classical null projection O' on S' . To leading order in small transverse displacements,

$$L_1 \approx L + \frac{(x'_1 - x)^2 + (y'_1 - y)^2}{2L}, \quad (5)$$

with an exactly similar expression for L_2 . Thus,

$$L_1 - L_2 \approx \frac{(x'^2_1 + y'^2_1) - (x'^2_2 + y'^2_2)}{2L} - \frac{x\Delta x' + y\Delta y'}{L}. \quad (6)$$

Define

$$\phi = \frac{\pi[(x'^2_1 + y'^2_1) - (x'^2_2 + y'^2_2)]}{\lambda L}, \quad (7)$$

corresponding to the phase difference $2\pi(OP_1 - OP_2)/\lambda$ between P_1 and P_2 , relative to the classical reference ray origin O . [This factor can be neglected for P_1 and P_2 sufficiently close to the reference ray OO' , ($OP_1 - OP_2$) $\ll \lambda$. This applies if the width of support of I is much larger than that of μ' , although it does not apply in all situations.] Then we find that the correlator on S' is, up to a normalization,

$$\begin{aligned} \mu'(\Delta x', \Delta y') &= e^{i\phi} \int_S dx dy I(x, y) \\ &\times \exp\left[\frac{-2\pi i(x\Delta x' + y\Delta y')}{\lambda L}\right]. \end{aligned} \quad (8)$$

This result is mathematically equivalent to the mutual coherence theory of van Cittert and Zernike, as described, for example, in [9]. In the quantum geometry interpretation, a spacetime state prepared with a distribution I on S produces a spacetime state with a wave function μ' on S' . The positions of events in each transverse direction on two geometrical wave fronts are (apart from the phase factor $e^{i\phi}$) Fourier transforms of each other. This wave optics relationship resembles that of conjugate variables in quantum mechanics. This can be seen most clearly if we assume that $\mu'(\Delta x')$ and $I(x)$ are Gaussians with respective standard deviations σ', σ . These are Fourier transforms of each other obeying Eq. (8) if

$$\sigma' \sigma = \lambda L. \quad (9)$$

The precise normalization adopted for mapping the optical model onto quantum geometry—the choice of the Planck length l_P as the fundamental wavelength λ , which sets the absolute scale for the holographic noise predictions below—derives from black hole physics and gravitational thermodynamics [1–8]. The hypothesis adopted here for the minimum wavelength or time of the quantum wave front geometry, $\lambda = l_P$, will be tested with measurements of holographic noise.

III. HOLOGRAPHIC NOISE IN INTERFEROMETERS

It appears that operational Michelson interferometric gravitational-wave detectors [23] are capable of measuring this form of quantum uncertainty of geometry. Let x denote the axis orthogonal to the incoming laser, and normal to the wave fronts incident on the dark port. In the language of the above analysis, the signal depends on the positions of two beamsplitter reflections in the x direction, described by distributions, $I(x)$ for one reflection and $\mu'(\Delta x')$ for the other. The measured intensity of the light gives the phase difference of the position at the two reflections, proportional to the difference in length between two arms of the interferometer. Holographic uncertainty in the relative positions at the two reflections creates noise in the measured signal, a random variation in the position of the beamsplitter. The positions are compared on wave fronts with normal separation of twice the arm length L_0 . Let σ and σ' denote the standard deviations of the x and x' position wave functions of the beamsplitter. We suppose that these obey the transverse position uncertainty relation [Eq. (9)], $\sigma' \sigma = 2l_P L_0$, since $2L_0$ is the separation between reflection events. The measured difference of arm lengths of the interferometer includes a sum of both variances, which is minimized when they are equal, so $\sigma \sigma' = \sigma^2 = \sigma'^2$. Sampled at times farther apart than the round trip travel time, the apparent difference in arm length determined by interferometry is a random variable with variance [14]

$$\Delta L_0^2 = \sigma^2 + \sigma'^2 = 2\sigma^2 = 4l_P L_0. \quad (10)$$

The variation causes a random shift in the detected phase signal that appears as measurable noise.

For the interferometric gravitational-wave detector GEO600 [24] the noise is quoted in terms of equivalent gravitational-wave metric strain, for a wave normally incident on the detector and polarization aligned with the arms for maximum signal. The power spectral density h^2 gives the variance in metric strain perturbation per frequency interval Δf . In GEO, the arms are $L_0 = 600$ m long and are folded once, so the time between the orthogonal beamsplitter position-measuring events is $4L_0/c$, corresponding to a resonant frequency $f_0 = c/4L_0$. The sum of the holographic variance in the two directions accumu-

lated over the resonance time is given by Eq. (10): $\Delta L_0^2 = 4l_P L_0$. The equivalent metric strain holographic spectrum h_H is defined by equating the holographic effect on the signal with that produced by a gravitational-wave spectrum h_H . The equivalent metric strain variance is the variance of one arm (in this case, half of ΔL_0^2) divided by the squared length of the laser path along one arm (in this case, $4L_0$): $\Delta L_0^2/2(4L_0)^2 \simeq h_H^2(f_0)\Delta f_0$. Setting the effective bandwidth $\Delta f_0 = f_0$ we obtain at $f_0 = 125$ kHz,

$$h_H(f_0) = \sqrt{t_P/2} = 1.6 \times 10^{-22} \text{ Hz}^{-1/2}. \quad (11)$$

This is a factor of $\sqrt{2}$ less than the somewhat cruder estimate in [14], but the basic result remains the same: at the resonance frequency, spacetime indeterminacy produces holographic noise in this interferometer with a power spectral density given in these units by half the Planck time. No parameters have been used in deriving Eq. (11). The numerical value assumes that the fundamental minimum time interval is the Planck time, $t_P \equiv l_P/c \equiv \sqrt{\hbar G_N/c^5} = 5 \times 10^{-44} \text{ Hz}^{-1}$. Measurements of holographic noise will provide a direct, precision test of this hypothesis.

The spectrum of equivalent h_H in the GEO600 signal below f_0 is approximately independent of frequency, down to the inverse residence time of a photon (about $f \simeq 550$ Hz). The flat spectrum arises because the entire laser light power cycles many times through the two arms and the beamsplitter. (GEO600 is more sensitive to holographic noise than LIGO [25], whose separate resonant arm cavities amplify the gravitational-wave effect on the signal but not the holographic noise.) Light reflected from one side of the beamsplitter at one time reaches the detected signal not just once but $\simeq f_0/f$ times. At frequency f , the signal phase measures an average arm difference over a time $\simeq 1/f$, and the multiple measurements reduce the error in the mean. As a result the holographic arm length difference variance goes like $\Delta L^2 \propto f$, and h_H is constant with frequency, with the value in Eq. (11).

The inverse residence time for light of about 550 Hz determines the sampling interval beyond which the measured arm differences become independent random variables. At lower frequency the total displacement in phase accumulates like a random walk, so $\Delta L^2 \propto t \propto f^{-1}$, and $h_H \propto \Delta L/f^{1/2} \propto f^{-1}$, or

$$\begin{aligned} h_H &\simeq \sqrt{t_P/2}(f/550 \text{ Hz})^{-1} \\ &= 1.6 \times 10^{-22}(f/550 \text{ Hz})^{-1} \text{ Hz}^{-1/2}, \\ f &< 550 \text{ Hz}. \end{aligned} \quad (12)$$

The only parameter in the predicted holographic noise spectrum in Eq. (12) is the value of the residence time, which has been set from the measured peak frequency of the optical gain in GEO600. The spatial and frequency

dependence of the noise, as well as its absolute level, are completely fixed by the holographic geometry hypothesis.

The predicted spectrum of holographic noise [Eqs. (11) and (12)] approximately agrees with the spectrum of an otherwise unexplained “mystery noise” currently measured [26] in GEO600 in its most sensitive band, $\approx 300\text{--}1400$ Hz, so it may already have been detected. If so, the sensitivity of GEO600 is already limited by space-time indeterminacy. Measurements of the noise spectrum will in that case provide a direct and precise measurement of the fundamental maximum frequency or minimum time

interval, independent of the conventional gravitational definition of the Planck time.

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