Addendum to the essay *Ordinary Analogues for Quantum Mechanics*

Commutator X P for the "Needle" analogy

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In quantum mechanics, when two conjugate operators describing observables (like position and momentum) operate consecutively on the state vector, the result of the operation depends on the order of the operators. In classical mechanics, the result would be independent of the order. This specific quantum-mechanical feature may be illustrated with the "needle/arrow analogy".

Let a travelling spinning needle be represented by the vector $|\Psi>$, see Fig. 1. When it is travelling in the x-direction, the needle rotates about its centre. The measure of this rotation is given by the derivative with respect to x. The x-coordinate of its centre is located at x_x. The x-coordinate of the centre of its derivative is located at x_p.

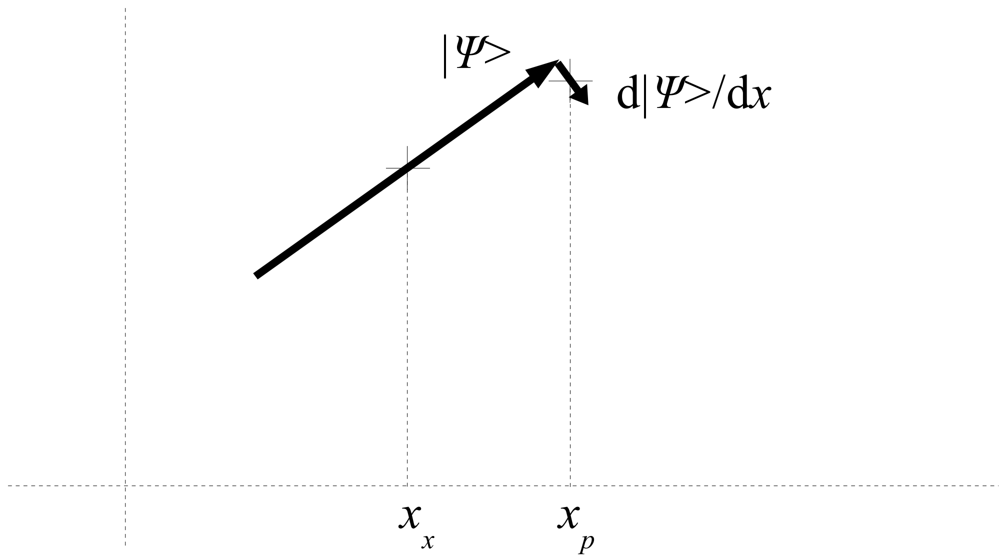

Figure 1 : x-coordinate of vectors representing the needle and its derivative with respect to x

Let **X** be the position operator assigning the value of the x-coordinate of the centre of the arrow representing the vector on which it operates and let **P** be the momentum operator differentiating the vector with respect to its x-coordinate. We then have the following relations:

$$\mathbf{P}\,\mathbf{X}\,|\Psi> \;=\; \mathbf{P}\,x_x\,|\Psi> = \mathrm{d}(x_x\,|\Psi>)\,/\,\mathrm{d}x = x_x\,\mathrm{d}|\Psi>\,/\,\mathrm{d}x \qquad (1)$$

$$\mathbf{X}\,\mathbf{P}\,|\Psi> \;=\; \mathbf{X}\,\mathrm{d}|\Psi>\,/\,\mathrm{d}x = x_p\,\mathrm{d}|\Psi>\,/\,\mathrm{d}x \qquad (2)$$

$$(\mathbf{X}\,\mathbf{P} - \mathbf{P}\,\mathbf{X})\,|\Psi> \;=\; (x_p - x_x)\,\mathrm{d}|\Psi>\,/\,\mathrm{d}x \qquad (3)$$

Equation (3) illustrates the fact that the commutator of operators acting on extended objects is not necessarily zero, unlike for idealized classical point particles.