Redshift in the Twin-Slit Experiment per RBW

1. Discrete Path Integral Formalism. We use graph theory since it has already been shown to provide an excellent mathematics for the construct of a discrete basis to quantum physics (e.g., Markopoulou & Smolin, 2004). We use our graphical approach to generate the kernel for a discrete transition amplitude, Z, for sources without scattering, i.e.,

$$Z = \int \ldots \int dQ_1 \ldots dQ_N \exp\left[\frac{i}{2}\vec{Q}\cdot\vec{\vec{A}}\cdot\vec{Q} + i\vec{J}\cdot\vec{Q}\right] \tag{1}$$

(Zee, 2003) whence

$$Z = \left(\frac{(2i\pi)^N}{\det(A)}\right)^{1/2} \exp\left[-\frac{i}{2}\vec{J}\cdot\vec{\vec{A}}^{-1}\cdot\vec{J}\right] \tag{2}.$$

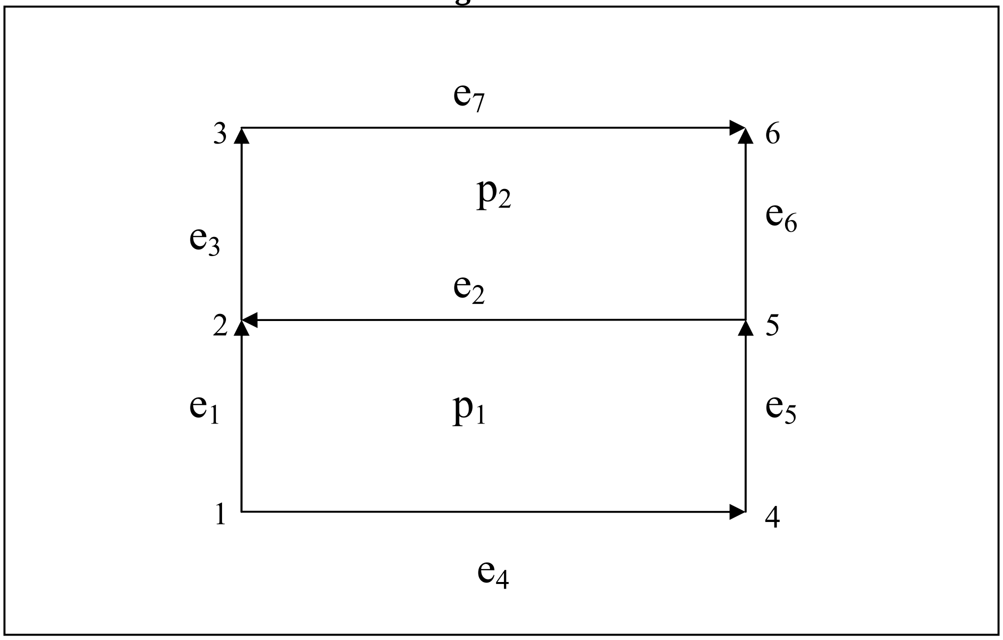
The matrix $\vec{\vec{A}}$ is the spatiotemporally discrete form of the differential operator in the action while $\vec{J}$ is the spatiotemporally discrete form of the 'sources'. Z can be viewed as a measure of the "symmetry" contained its kernel or *actional*

$$\Sigma = \frac{1}{2}\vec{\vec{A}} + \vec{J} \tag{3}$$

which yields the discrete action after operating on a particular vector $\vec{Q}$, e.g., plugging a different path x(t) into the Lagrangian produces a different action. That is, the action is the *range* of the actional, which represents a *fundamental, invariant description of the experimental arrangement in the computation of* Z. For this reason, we call Z the *symmetry amplitude* of the spatiotemporal experimental configuration (to include the outcomes), where "measure of symmetry" is understood broadly as the coherence of stationary points in the phase of Z.

We will consider a simple graph with six vertices, seven links and two plaquettes (cells) for our two-source model (Figure 1). We begin by constructing the boundary operators over our graph.

Figure 1



The boundary of $\mathbf{p}_1$ is $\mathbf{e}_4 + \mathbf{e}_5 + \mathbf{e}_2 - \mathbf{e}_1$, which also provides an orientation. The boundary of $\mathbf{e}_1$ is $\mathbf{v}_2 - \mathbf{v}_1$, which likewise provides an orientation. Using these conventions for the orientations of links and plaquettes we have the following boundary operator for $C_2 \rightarrow C_1$, i.e., space of plaquettes mapped to space of links in the spacetime chain complex:

$$\partial_2 = \begin{bmatrix} -1 & 0 \\ 1 & -1 \\ 0 & -1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \tag{4}$$

Notice the first column is simply the links for the boundary of $\mathbf{p}_1$ and the second column is simply the links for the boundary of $\mathbf{p}_2$. We have the following boundary operator for $C_1 \rightarrow C_0$, i.e., space of links mapped to space of vertices in the spacetime chain complex:

$$\partial_1 = \begin{bmatrix} -1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \tag{5}$$

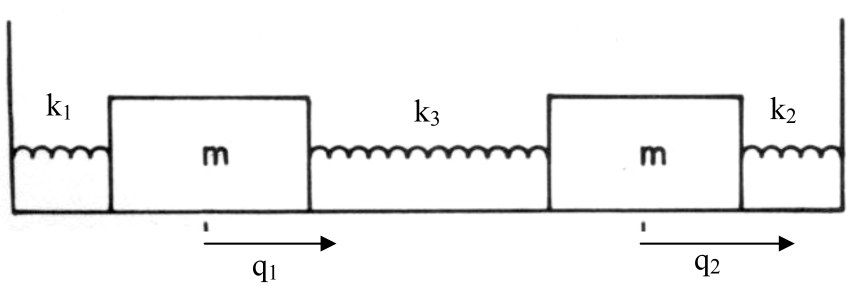
which completes the spacetime chain complex, $C_0 \xleftarrow{\partial_1} C_1 \xleftarrow{\partial_2} C_2$. Notice the columns are simply the vertices for the boundaries of the edges. These boundary operators satisfy $\partial_1\partial_2 = 0$ as required for "boundary of a boundary is zero," in accord with divergence-free sources (Misner *et al.*, 1973). We want our self consistency criterion (SCC) ultimately founded on this topological maxim so we construct our actional from the boundary operators of our spacetime chain complex. The manner by which we do this is suggested by the discrete action for coupled harmonic oscillators on our simple graph.

The potential for coupled oscillators can be written

$$V(q_1,q_2) = \sum_{a,b} \frac{1}{2} k_{ab} q_a q_b = \frac{1}{2} k q_1^2 + \frac{1}{2} k q_2^2 + k_{12} q_1 q_2 \tag{6}$$

where $k_{11} = k_{22} = k$ (positive) and $k_{12} = k_{21}$ (negative) per the classical analogue (Figure 2)

Figure 2



with $k = k_1 + k_3 = k_2 + k_3$ and $k_{12} = –k_3$ to recover the form in Eq. (6). The Lagrangian is then

$$L = \frac{1}{2}m\dot{q}_1^2 + \frac{1}{2}m\dot{q}_2^2 - \frac{1}{2}kq_1^2 - \frac{1}{2}kq_2^2 - k_{12}q_1q_2 \tag{7}$$

so our QM symmetry amplitude is

$$Z = \int Dq(t)\exp\left[-\int_0^T dt\left[\frac{1}{2}m\dot{q}_1^2 + \frac{1}{2}m\dot{q}_2^2 + V(q_1,q_2) - J_1q_1 - J_2q_2\right]\right] \tag{8}$$

after Wick rotation. This gives

$$\vec{\vec{A}} = \begin{bmatrix} \left(\frac{m}{\Delta t} + k\Delta t\right) & \frac{-m}{\Delta t} & 0 & k_{12}\Delta t & 0 & 0 \\ \frac{-m}{\Delta t} & \left(\frac{2m}{\Delta t} + k\Delta t\right) & \frac{-m}{\Delta t} & 0 & k_{12}\Delta t & 0 \\ 0 & \frac{-m}{\Delta t} & \left(\frac{m}{\Delta t} + k\Delta t\right) & 0 & 0 & k_{12}\Delta t \\ k_{12}\Delta t & 0 & 0 & \left(\frac{m}{\Delta t} + k\Delta t\right) & \frac{-m}{\Delta t} & 0 \\ 0 & k_{12}\Delta t & 0 & \frac{-m}{\Delta t} & \left(\frac{2m}{\Delta t} + k\Delta t\right) & \frac{-m}{\Delta t} \\ 0 & 0 & k_{12}\Delta t & 0 & \frac{-m}{\Delta t} & \left(\frac{m}{\Delta t} + k\Delta t\right) \end{bmatrix} \tag{9}$$

on our graph. Thus, we borrow (loosely) from Wise (2006) and suggest $\vec{\vec{A}} \propto \partial_1\partial_1^+$ since

$$\partial_1\partial_1^+ = \begin{bmatrix} 2 & -1 & 0 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ 0 & -1 & 2 & 0 & 0 & -1 \\ -1 & 0 & 0 & 2 & -1 & 0 \\ 0 & -1 & 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 0 & -1 & 2 \end{bmatrix} \tag{10}$$

produces precisely the same form as Eq. (9) and quantum theory is known to be "rooted in this harmonic paradigm" (Zee, 2003). [In fact, these matrices will continue to have the same form as one increases the number of vertices in Figure 1.] Now we construct a suitable candidate for $\vec{J}$, relate it to $\vec{\vec{A}}$ and infer our SCC.

Recall that $\vec{J}$ has a component associated with each node so here it has components, J_n, n = 1, 2, …, 6; J_n for n = 1, 2, 3 represents one 'source' and J_n for n = 4, 5, 6 represents another 'source'. We propose $\vec{J} \propto \partial_1\vec{e}$,where e_i are the links of our graph, since

$$\partial_1\vec{e} = \begin{bmatrix} -1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ e_7 \end{bmatrix} = \begin{bmatrix} -e_1 - e_4 \\ e_1 + e_2 - e_3 \\ e_3 - e_7 \\ e_4 - e_5 \\ -e_2 + e_5 - e_6 \\ e_6 + e_7 \end{bmatrix} \qquad (11)$$

provides a means of understanding vertices in terms of links and ultimately we want sources defined relationally. For example, vertex 1 is the origin of both links 1 and 4 (negative/positive means the link starts/ends at that vertex) and the first entry of $\partial_1\vec{e}$ is $-e_1 - e_4$. Since J_n are associated with the vertices to represent 'things', $\vec{J} \propto \partial_1\vec{e}$ is a graphical representation of "relata from relations." [Note: $\partial_1\vec{e}$, which we denote $\vec{v}\,*$ and associate with $\vec{v}$, is not equal to $\vec{v}$ proper as will be seen below.]

With these definitions of $\vec{\vec{A}}$ and $\vec{J}$ we have, *ipso facto*, $\vec{\vec{A}}\vec{v} \propto \vec{J}$ as the basis of our SCC since

$$\partial_1\partial_1^{+}\vec{v} = \begin{bmatrix} 2 & -1 & 0 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ 0 & -1 & 2 & 0 & 0 & -1 \\ -1 & 0 & 0 & 2 & -1 & 0 \\ 0 & -1 & 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} = \begin{bmatrix} 2v_1 - v_2 - v_4 \\ -v_1 + 3v_2 - v_3 - v_5 \\ -v_2 + 2v_3 - v_6 \\ -v_1 + 2v_4 - v_5 \\ -v_2 - v_4 + 3v_5 - v_6 \\ -v_3 - v_5 + 2v_6 \end{bmatrix} = \begin{bmatrix} -e_1 - e_4 \\ e_1 + e_2 - e_3 \\ e_3 - e_7 \\ e_4 - e_5 \\ -e_2 + e_5 - e_6 \\ e_6 + e_7 \end{bmatrix} = \partial_1\vec{e} = \vec{v}\,* \qquad (12)$$

where we've used $e_1 = v_2 - v_1$ (etc.) to obtain the last column, which constitutes a definition of links in terms of vertices. It is clear that according to this definition of links in terms of vertices, $\vec{v} \neq \vec{v}\,*$ ($\partial_1\partial_1^{+}$ is not the identity matrix). In fact we have

$$\partial_1^{+}\vec{v} = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} = \begin{bmatrix} v_2 - v_1 \\ v_2 - v_5 \\ v_3 - v_2 \\ v_4 - v_1 \\ v_5 - v_4 \\ v_6 - v_5 \\ v_6 - v_3 \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ e_7 \end{bmatrix} = \vec{e} \tag{13}$$

which reflects the statement *supra*, “Notice the columns are simply the vertices for the boundaries of the edges.” Thus, the SCC $\vec{\vec{A}}\vec{v} \propto \vec{J}$ obtains tautologically via the maxim, “the boundary of a boundary is zero,” as desired.

Using $\vec{J} = \alpha\partial_1\vec{e}$ and $\vec{\vec{A}} = \beta\partial_1\partial_1^{+}$ with the SCC gives $\vec{\vec{A}}\vec{v} = \frac{\beta}{\alpha}\vec{J}$, so that $\vec{v} = \frac{\beta}{\alpha}\vec{\vec{A}}^{-1}\vec{J}$. However, $\vec{\vec{A}}^{-1}$ doesn’t exist because $\vec{\vec{A}}$ is singular, which means of course that Eq. (1) is ill-defined for this problem. The reason $\vec{\vec{A}}$ is singular is because one of its eigenvalues is zero and that obtains because $\vec{\vec{A}}$ is a difference matrix whose rows are vectors spanning an (N-1)-dimensional hyperplane of the N-dimensional vector space. The eigenvector with eigenvalue of zero is normal to this hyperplane (which you can see is [1,1,1,…,1] since $\sum_j A_{ij} = 0$), so we propose a discrete ‘renormalization’ of Eq (1) whereby the integral is restricted to the hyperplane containing the vectors of $\vec{\vec{A}}$, i.e.,

$$Z = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} d\widetilde{Q}_1 \dots d\widetilde{Q}_{N-1} \exp\left[\sum_{j=1}^{N-1}\left(\frac{i}{2}\widetilde{Q}_j^2 a_j + i\widetilde{J}_j\widetilde{Q}_j\right)\right] \tag{14}$$

where $\widetilde{Q}_j$ are the coordinates associated with the eigenbasis of $\vec{\vec{A}}$ and $\widetilde{Q}_N$ is associated with eigenvalue zero, a_j is the eigenvalue of $\vec{\vec{A}}$ corresponding to $\widetilde{Q}_j$, and $\widetilde{J}_j$ are the components of $\vec{J}$ in the eigenbasis of $\vec{\vec{A}}$. Again, on our view, Z does not reflect a “sum over all paths in configuration space,” but rather it is a ‘mathematical machine’ which produces a relative symmetry measure of the various Σ associated with different experimental outcomes and configurations. Since $\vec{J}$ resides in this (N-1)-dimensional hyperplane as well (which you can see from $\sum_i J_i = 0$), restricting the integral in Eq. (1) to the hyperplane spanned by the vectors of $\vec{\vec{A}}$ is not at all unreasonable. This 'renormalization' revises Eq. (2) to read

$$Z = \left(\frac{(2i\pi)^{N-1}}{\prod_{j=1}^{N-1} a_j} \right)^{1/2} \prod_{j=1}^{N-1} \exp\left[-\frac{i}{2} \frac{\tilde{J}_j^2}{a_j} \right] \qquad (15).$$

Since $\vec{J}$ is defined via links we have characterized the symmetry amplitude in terms of relations and the non-zero eigenvalues of $\vec{\vec{A}}$.

For general N, half the eigenvectors of $\vec{\vec{A}}$ are of the form $\begin{bmatrix} \vec{x} \\ \vec{x} \end{bmatrix}$ and half are of the form $\begin{bmatrix} \vec{x} \\ -\vec{x} \end{bmatrix}$. The eigenvalues are given by $\lambda \pm 1$ where $\lambda - 1$ is the eigenvalue for $\begin{bmatrix} \vec{x} \\ \vec{x} \end{bmatrix}$, $\lambda + 1$ is the eigenvalue for $\begin{bmatrix} \vec{x} \\ -\vec{x} \end{bmatrix}$, and $\lambda_j = 3 - 2\cos\left(\frac{j2\pi}{N}\right),\ \ j = 0,\ldots,\left(\frac{N}{2} - 1\right)$. The k components of $\vec{x}$ for a given λ_j are $x_{jk} = \sqrt{\frac{2}{N}}\cos\left(\frac{j(2k-1)\pi}{N}\right),\ \ k = 1,\ldots,\frac{N}{2}$ for $j > 0$ and $x_{0k} = \sqrt{\frac{1}{N}},\ \ k = 1,\ldots,\frac{N}{2}$ for $j = 0$ ($j = 0 \rightarrow$ eigenvalues of $\vec{\vec{A}}$ are 0 and 2). We have N nodes and (N + N/2 – 2) links. Define the temporal (vertical) links e_i in terms of vertices v_i in the following fashion:

$$e_i = v_{i+1} - v_i \qquad \text{i = 1 to N/2 – 1}$$

and

$$e_{\frac{N}{2}+i-1} = v_{\frac{N}{2}+i+1} - v_{\frac{N}{2}+i} \qquad \text{i = 1 to N/2 – 1.}$$

Define the spatial (horizontal) links as follows:

$$e_{N+i-2} = v_{\frac{N}{2}+i} - v_i \quad \text{i = 1 to N/2.}$$

This graph and the above definition of $\vec{J}(e_i)$ give us

$$\vec{J} = \begin{bmatrix} -e_1 - e_{N-1} & \\ -e_i + e_{i-1} - e_{N+i-2} & i = 2,...\frac{N}{2}-1 \\ e_{\frac{N}{2}-1} - e_{N+\frac{N}{2}-2} & \\ e_{N-1} - e_{\frac{N}{2}} & \\ e_{\frac{N}{2}+i-2} + e_{N+i-2} - e_{\frac{N}{2}+i-1} & i = 2,...\frac{N}{2}-1 \\ e_{N+\frac{N}{2}-2} + e_{N-2} & \end{bmatrix} \qquad (16).$$

We then need to find the projection of $\vec{J}$ on each of the orthonormal eigenvectors of $\vec{\vec{A}}$ that have non-zero eigenvalues. Call each projection $\widetilde{J}_i = \langle i|J\rangle$, where $\langle i|$ is the i$^{\text{th}}$ orthonormal eigenvector. Let a_i (i = 1, N-1) be the non-zero eigenvalues of $\vec{\vec{A}}$ associated with the eigenvectors $\langle i|$, (i = 1, N-1). Then we need to compute

$$\Phi = -\sum_{i=1}^{N-1} \frac{\left(\widetilde{J}_i\right)^2}{2a_i} \qquad (17).$$

2. Redshift in the Twin-slit Experiment. For a free particle of mass m we have (Shankar, 1994)

$$\psi = \alpha\sqrt{\frac{m}{2\pi\hbar it}}\exp\left[\frac{imx^2}{2\hbar t}\right] \propto \exp\left[\frac{imx^2 t}{2\hbar t^2}\right] = \exp\left[\frac{imv^2 t}{2\hbar}\right] = \exp\left[\frac{ipvt}{2\hbar}\right] = \exp\left[\frac{iv_\varphi t}{\lambda}2\pi\right] \qquad (18)$$

where v_φ is the phase velocity and equal to half the particle velocity (Park, 1992). So, in the twin-slit case the interference pattern is given by

$$|\psi_1 + \psi_2|^2 \propto \left|\exp\left[\frac{iv_\varphi t_1}{\lambda}2\pi\right] + \exp\left[\frac{iv_\varphi t_2}{\lambda}2\pi\right]\right|^2 = 2 + 2\cos\left[\frac{v_\varphi(t_1 - t_2)}{\lambda}2\pi\right] \qquad (19)$$

and therefore maxima occur at angles where

$$v_\varphi(t_1 - t_2) = n\lambda \quad n \in \mathbb{Z} \qquad (20).$$

For photons

$$\psi \propto \exp\left[\frac{iEt}{\hbar}\right] = \exp\left[\frac{ihft}{\hbar}\right] = \exp\left[\frac{ict}{\lambda}2\pi\right] \tag{21}$$

so maxima occur at angles where

$$c(t_1 - t_2) = n\lambda \quad n \in \mathbb{Z} \tag{22}$$

[Aside: Since a photon yields a single click (not a series of clicks whence a trajectory), c cannot be measured for a photon just as v_φ cannot be measured for a massive particle. In the twin-slit experiment, there are no trajectories (sets of clicks lending themselves to trans-temporal identification), so we will simply use one expression containing v when referring to the QM result.]

The RBW result for the phase Φ of ψ, Eq. (17), is given in three parts, i.e., $\Phi = -(\Phi_S + \Phi_T + \Phi_{ST})/2$, where

$$\Phi_S = \frac{2}{N}\left[\sum_{k=1}^{\frac{N}{2}-1} e_{k+N-2}\right]^2 \tag{23}$$

involves only spatial links,

$$\Phi_T = \sum_{j=1}^{\frac{N}{2}-1}\frac{2}{N}\left[\sum_{k=1}^{\frac{N}{2}-1}\left(e_k + e_{k+\frac{N}{2}-1}\right)\sin\left(\frac{jk2\pi}{N}\right)\right]^2 \tag{24}$$

involves only temporal links, and

$$\Phi_{ST} = \sum_{j=1}^{\frac{N}{2}-1}\frac{4}{N\left(1+\sin^2\frac{j\pi}{N}\right)}\left[\sin\frac{j\pi}{N}\sum_{k=1}^{\frac{N}{2}-1}\left(e_k - e_{k+\frac{N}{2}-1}\right)\sin\left(\frac{jk2\pi}{N}\right) + \sum_{k=1}^{\frac{N}{2}}\left(e_{k+N-1}\right)\cos\left(\frac{(2k-1)j\pi}{N}\right)\right]^2 \tag{25}$$

involves a mix of temporal and spatial links. We will need

$$\Phi = \left[\frac{vt}{\lambda} 2\pi \right] \tag{26}$$

where *v* may be viewed as a scaling factor linking space and time. In the twin-slit experiment this means

$$\Phi_1 - \Phi_2 = \frac{v(t_1 - t_2)}{\lambda} \tag{27}.$$

Let e_i be the links of graph 1 (whence Φ_1) and $\widetilde{e}_i$ the links of graph 2 (whence Φ_2). We expect the temporal links at the click to be equal between graphs since these links represent one and the same click. We also expect the temporal links at each slit to be equal since they are connected to one and the same source, i.e., they're coherent. Suppose further that all temporal links of either graph are equal to one another, so we have $\widetilde{e}_i = e_i = e_T$ for *i* = 1 to N – 2. We do expect the spatial links to differ between graphs, reflecting the different distances from each slit to a particular click location. Let us assume all spatial links of either graph are equal to one another (static situation) so we have $e_{N+i-2} = e_x$ and $\widetilde{e}_{N+i-2} = \widetilde{e}_x$ for *i* = 1 to N/2. In this case, we find $\Phi_{ST} = 0$ and

$$\Phi_S + \Phi_T = \frac{N}{2} e_x^2 + (N-2) e_T^2$$

Since N/2 is the number of spatial links and N – 2 is the number of temporal links, this is the result that would've obtained if $\vec{\vec{A}}^{-1}$ existed; $\vec{v} = \vec{\vec{A}}^{-1}\vec{J} \rightarrow \vec{J} \cdot \vec{\vec{A}}^{-1}\vec{J} = \vec{J} \cdot \vec{v} = \sum_{All} e_i^2$. We now have

$$\frac{v(t_1 - t_2)}{\lambda} = \frac{N}{4}\left(e_x^2 - \widetilde{e}_x^2\right) \tag{28}$$

(here and after we drop the irrelevant negative sign in Φ). Let the links subsume the scaling factor *α* for $\vec{J}$ and let each eigenvalue of $\vec{\vec{A}}$ be multiplied by its scaling factor *β*, then Eq. 28 is revised

$$\frac{v(t_1 - t_2)}{\bar{\lambda}} = \frac{N}{4\beta}\left(e_x^2 - \tilde{e}_x^{\,2}\right) \qquad (29).$$

This is promising since it has the right general form, although the parameters on the RHS are, as yet, underdetermined.

Now suppose the spatial links increase/decrease by δ from one to the next, i.e., $e_{i+1} = e_i \pm \delta$ for the spatial links. We find that Eq. (29) is now given by

$$\frac{v(t_1 - t_2)}{\bar{\lambda}} = \frac{N}{4\beta}\left[\left(e_x^2 - \tilde{e}_x^2\right) \quad \pm \quad \left(\frac{N}{2} - 1\right)\left(e_x - \tilde{e}_x\right)\delta\right] \qquad (30).$$

A *decrease* in phase is consistent with an *increase* in λ (redshift) per QM, i.e., LHS of Eq. (30). *Prima facie* this is backwards from our graphical depiction, since our phase increases as the spatial link size increases and we might naively assume that increasing spatial link size corresponds to increasing spatial distance, thereby producing a redshift and a *decrease* in phase. However, our result makes sense if the spatial links of our graph depict *internal* spatial distance so that larger spatial link size represents more *intrinsic* units of measure between sources, i.e., more λ. To fit more waves in a given extrinsic spatial distance means the waves are blueshifted, so *increasing* spatial link size in RBW corresponds to *blueshifted* λ in QM (and *decreasing* spatial link size corresponds to *redshifted* λ).

The phase difference $\frac{v(t_1 - t_2)}{\bar{\lambda}}$ is just 2π times the number of waves n in the path length difference between the slits to a particular point on the detector. If we obtain what we believe to be two equivalent spectra with different distributions on the detector, we attribute it to $\lambda \rightarrow \lambda'$. One of the two spectra is then said to be a redshifted version of the other, where redshift is quantified by $z \equiv \frac{\lambda'}{\lambda} - 1$. If we characterize the path length difference by the number of waves n or n' in each case, then we have $z = \frac{n}{n'} - 1$. For the two cases given by Eqs. (29) & (30) we have

$$n2\pi = \frac{N}{4\beta}\left(e_x^2 - \tilde{e}_x^{\,2}\right)$$

and

$$n^{/}\,2\pi = \frac{N}{4\beta}\left[\left(e_x^2 - \tilde{e}_x^{\,2}\right) \quad \pm \quad \left(\frac{N}{2} - 1\right)\left(e_x - \tilde{e}_x\right)\delta\right]$$

so that

$$z = \frac{\mp\left(\frac{N}{2} - 1\right)\left(e_x - \tilde{e}_x\right)\delta}{\left(e_x^2 - \tilde{e}_x^{\,2}\right) \quad \pm \quad \left(\frac{N}{2} - 1\right)\left(e_x - \tilde{e}_x\right)\delta} \tag{31}$$

for blueshift/redshift respectively.