A still valid argument by Ritz (version 09, to be amended)

Eckard Blumschein

Abstract

Future events cannot influence the past. Accordingly, only data from the past can be measured. When physics predicts what is expected to become past, then it may be considered as restricted to future elapsed time. So far, physics ignores this restriction on the expense of loosing realism and simplicity. The restriction to at least expectedly elapsed time yields surprising revelations when compared with the traditional tense-less point of view: The usual complex Fourier transform of an unrelated quantity is twice burdened with redundancy being required due to an arbitrarily chosen reference of time. Misinterpretation of the redundant data was suspected and verifiable shown to be responsible for oddities in physics.

1. Introduction: Really outside the realm of science?

An argument by W. Ritz is still valid: Future events cannot influence the past. In 1909, he agreed not to agree with Albert Einstein [1] who later reiterated his position [2]:

> *Einstein wrote: For us believing physicists, the division into past, present and future has merely the meaning of an albeit obstinate illusion." … He confessed that the problem of the Now worried him seriously. He explained that the experience of the Now means something special for man, something essentially different from the past and the future, but that this important difference does not and cannot occur within physics. That this experience cannot be grasped by science seemed to him a matter of painful but inevitable resignation. So he concluded that there is something essential about the Now which is just outside the realm of science.*

Carl-Friedrich von Weizsaecker in [3] also failed to find an answer. Werner Heisenberg reported [4] that Einstein argued:

> *But you don't believe seriously that only observable quantities must go into a physical theory? … It is the theory that decides what we can observe.*

Is space-time something one can move within back and forth? No. Future time can definitely not be measured in advance. Likewise, we may abandon the belief in anything to come for sure. Reality restricts to ongoing effects of what happened. This is quite obvious in spectral analysis where only past data are available. Clocks exclusively count positive elapsed time. Consequently elapsed time is the primary measure; the now is the only natural reference point. One has anyway to admit: Negative values of time can be avoided in general with an appropriate shift of the point of reference. Correspondingly it is possible to adequately replace complex-valued functions of -oo<t<oo by real functions of oo>τ=$t_{elapsed}$>0 [5]. Of course, someone who merely intends prediction of future quantities instead of analyzing past ones will not appreciate this option. The argument by Ritz is indeed irrelevant on condition there is no compelling link to reality but only an artificial set of assumed inputs. However, beware of the belief in determinacy since there are no absolutely closed systems in reality! While the laws of physics are perhaps valid forever, the state of any actual or anticipated physical process is the sum of possibly unseen influences exclusively belonging to earlier events having never a negative distance from the now. All physics is causal in that sense. Causes result in effects, not vice versa. Seemingly cyclical processes have directions, too: Chickens lay eggs. Even Joseph Fourier's symmetrical heat-conducting ring does not give rise to deny a directed time-scale. Those like David Hilbert who questioned the so called flow of time or at least partially its arrow [6] or tried to interpret it as an effect of psychology are proponents of a physics which is based on excessively generalizing of mathematical tenets, fitted to the idea of an a priori existing space-time. They convey Georg Cantor's populist attitude into physics: "The essence of mathematics is just its freedom" [7].

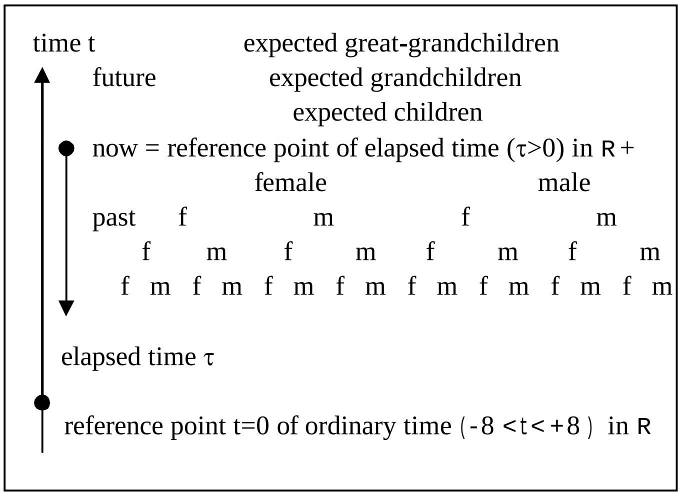

Fig. 1. *Family tree with upward gliding zero of elapsed time*

Science has to respect the now as the border of what got history. Fig. 1 illustrates how the now glides relative to the sequence of events. Processes integrate influences. Analog computers were based on integrators. Differential equations (DEQs) do not describe the primary relations. Since singularized and even personalized causes were much easier imaginable to our ancestors than a plurality of interrelated influences, physical theory still suffers from a very old habit: It abstracts from own perspective and shifts the forward-looking point of view to a suspected former cause. Pierre Laplace shared this shifted perspective with the story of Adam and Eva. Creator's perspective is also implied in notions like Big Bang and causal signal.

DEQs only fit on the more abstract level where reality, including the arrow of time, has been stripped off. Disliking this restriction, mathematicians like Laurent Schwartz [8] formally removed it by means of generalized functions that can be differentiated at will. He did of course not change reality. This paper prefers an alternative mathematics that obeys natural peculiarities of reality. Let’s consider functions of arguments that are the product of two quantities like elapsed time τ and circular frequency ω or radius r and wave number k_r whose units are reciprocal to each other.[1] All these quantities do not change their sign. We do not need real numbers of IR with both signs but only IR+ without sign. Mathematicians are ascribing no support to the redundant ‘half’-plane while still footing on IR. Because the exemplary chosen variable τ in Fig. 2 is restricted to values without sign, the messy diversity of so called singularity functions reduces to an ordered set with only one singularity each.

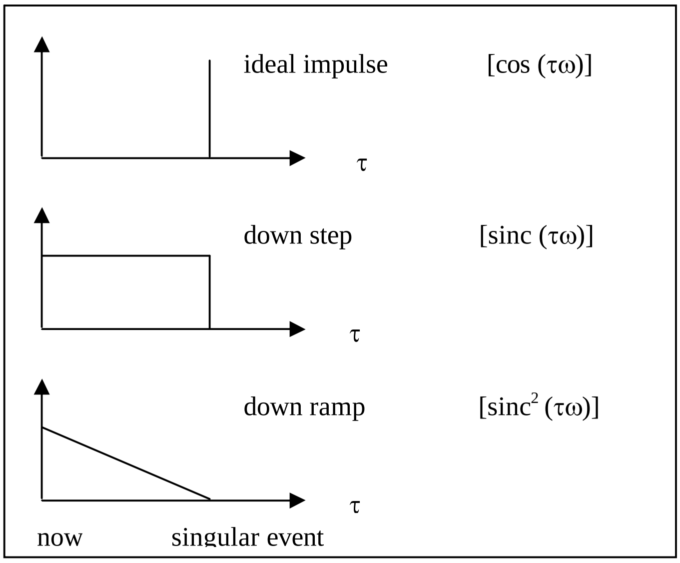

Fig. 2 *The first three singularity functions of elapsed time. In parentheses their cosine transform where τ is parameter.*

Consider integration and differentiation proceeding from singular event to now. Then all integrals converge. Because all integrals start with the initial value zero, there are no constants to be added to integrals and no ones lost with differentiation. Integration of the ideal impulse yields a down step. Integration of the latter yields a down ramp, and so on without limit. Down step and down ramp replace rectangular and triangular, respectively, pulses that have an additional singularity in the non-existing future. With Galileo Galilei’s notion of absolute infinity, the ideal impulse cannot be further differentiated. Ideal doublets combine ideal impulses.

Functions of τ can be approximated by superimposed singularity functions. Fig. 4 shows that the cosine transform (CT) of $f(\tau)$ can be calculated as the sum of belonging CTs of all constituting points, steps or ramps.

Restricting to IR+ is still in its infancy, notwithstanding the existence of almost forgotten not yet generalized tools like Duhamel’s integral from 0 to oo instead of convolution from -oo to +oo. So far, only a few mathematicians [9] consider rigs the more fundamental concept as compared to rings. Einstein was not wrong when he envisioned something outside the established science. R+ has been despised for centuries at least in theory, though CT got applied for audio coding. Usual timescale and complex Fourier transformation (FT) do and will further dominate science and technology.

Elapsed time and CT must nonetheless be considered as non- redundant special cases, just tailor-made for reality.

2. A Dilemma with redundancy due to arbitrary reference

Because any realistic function of elapsed time $f(\tau)$ is fully represented with τ in IR+, it can immediately be transformed into a function of frequency by means of the CT, whereas the complex FT would demand the preparatory measure of analytic continuation, usually performed by means of appending a zero-valued future. FT then requires one more arbitrary step: One has to split the combination of the original function and the appended nothing into two orthogonal components, one has even and the other one odd symmetry with respect to the argument zero. Finally the integration extends over the unreal range from minus to plus infinity. Odd components mutate into imaginary parts of FT. Inverse FT correctly returns $f(\tau)$ and its zero-valued appendix.

If a function f(x) only extends between x_1 and x_2 and exhibits even symmetry with respect to a point $(x_1+x_2)/2$, then one may prefer simply shifting it in order to reach symmetry with respect to this point instead of continuation with split zeros. In this case, FT yields a purely real result as normally does CT. In general, so-called linear phase and imaginary part depend at will on a shift relative to the defined origin.

Did someone miss the opportunity to introduce elapsed time in physics? When René Descartes (1596-1650) created spatial coordinates, negative numbers were new. He hesitated to use negative coordinates, and it was left to Newton to point out their usefulness [10]. The distinction between years BC and AD was already made, and nobody could imagine another time scale than the usual one. When Fourier formulated his harmonic analysis, he also did not have any chance but to stick on the ordinary event-related notion of time. Remarkably, he wrote at least once an integral with a lower limit 0 instead of minus infinity. One needs an absolute reference of a quantity if one intends to attribute exclusively positive arguments to it. In case of time, such natural zero is available if one does no longer refer the event of concern to an arbitrary reference event but to the now. While the now is gliding on the traditional scale, it is the only imaginable absolute and therefore natural reference point. Who disdains it is forced to arbitrarily choose a reference point. In the Christian world the standard reference is given by the birth of Christ and midnight at New Year in Greenwich.

For the sake of an efficient theory, the zero t=0 of the used time scale is often shifted to the center of something symmetrical, e.g., to the middle of a Gaussian impulse or between the past and its mirror picture.

The appended zeroes are redundant. Of course, there are not more positive and negative numbers together as compared to the amount of natural numbers. However, a finite set of data for sign-less argument becomes twice as large if one

[1] Considering pairs like ω and τ together avoids trouble with the unit of $\delta(t)$ due to the usual definition that presupposes an area of the size one.

appends zero-valued data for negative arguments. Since FT is a complex integral transform, it doubles the volume of data once again without incorporating new information. Its chosen kernel arbitrarily replaces 2 cos(x) = exp(ix) + exp(-ix) by either the first or the second term alone. Notice: exp(ix) = cos(x)+i sin(x). Accordingly, size and sign of the imaginary part depend on two choices: reference point and replacement. Imaginary parts vanish with a correct inverse transform. They must not be lost before the original one-sidedness is decoded from the fourfold redundancy.

Temperature, length, mass, energy, entropy, elapsed time, virtually all basic physical quantities can be reduced to a one-sided ones with an absolute zero. CTs for functions of such arguments are likewise real and never negative, for instance, the function of wave-number k_r belonging to a function of radius r.

The same distance r can be ascribed to a position x in a coordinate system with arbitrarily chosen reference where x can be negative. In this case, one needs a FT, and yields a complex function of positive as well as negative wave numbers k_x. Such result is twice redundant and twice unrealistic: Reality is real, and negative wave numbers do not have a direct correlate in reality.

Alternatively, we may choose the function f(k) of the physically correct one-sided real wave number k as the input of a FT. This time, we arrive at the strange result of a complex function F(r) of a positive as well as negative radius. The dilemma is obvious. Complex Fourier transformation always links a physically correct original with something complex that has to be interpreted with care:

Matrices expressing F(r) or F(k), respectively, in complex domain must be Hermitian, i.e., equal to its conjugate transpose. This means anti-symmetry of the imaginary part, which stands for a one-sided original function, i.e., a function of positive real argument. The Hermitian matrix with complex entries does not correspond to a complex but to a real FT counterpart. Be not mislead by the fact that complex domain is the mathematically more general one as compared to the real domain. The degrees of freedom are restricted by physics.

Among the most frequent consequences of mistaken complex representations were non-causalities, unnecessarily redundant data, and the obvious failure to adequately mimic the observed frequency analysis within cochlea.

Many experts worried a lot about negative frequency. Some of them tried and a very few are still trying to interpret the complex representation as if it was a real one. Common sense told the physicists of the past century that frequency is always a positive quantity. The ‘unphysical’ quantities of complex domain must not be corrected but comprehensively interpreted, instead! Negative frequency just reflects the fact that future events are not yet reality. This insight might hopefully clear up some fallacies for good, in particular those behind allegedly measured superluminal propagation of signals [11]. Only the suggested above naked CT in $\mathbb{R}+$ could radically avoid from the very beginning the enforced redundancy being responsible for the dilemma and for various mistakes.

3. Improper interpretation of formal mirror-symmetry

If an alleged symmetry is perfect, then it is possibly based on the mistake of attributing reality to the formal even and odd symmetries of the real and the imaginary part, respectively. Because genuine symmetries are rarely flawless, fallacies were suspected and always confirmed in detail except for one case [12] of definitely wrong results. Perfect T-symmetry in quantum mechanics nurtures the suspicion that this type of misinterpretation may affect quantum mechanics, too.

Why did apparently nobody feel responsible to express his awareness of the above-mentioned dilemma? Many experts agree that the wave function is complex and the Hamiltonian is real. The reason for this arbitrary but not occasional choice seems to be obvious: Heisenberg, Erwin Schroedinger and also Paul A. M. Dirac used the same trick and started their reasoning in the real domain not as usual at an assumed functions of time but at what was measurable in terms of a frequency spectrum, i.e., at functions of energy or frequency, respectively. Schroedinger definitely ignored the one-sidedness of reality when he supposed in [13]:

> … *one may consider the real part of Ψ as the real wave function, if necessary.*

He returned from complex domain into the real one just by multiplying Ψ with its complex conjugate Ψ*. This quite common method was formally flawless from Einstein’s perspective while it is insufficient if one is ready to accept Ritz’s argument.

Ideal functions like sin(ωt) alone do not at all fit to the physical reality: They extend symmetrically from minus infinity to plus infinity while reality only exists for elapsed time. With usual time, adequate decoding is a must. Not just Schroedinger, the whole community failed to realize that only the elapsed half of the real part of Ψ must be considered the really real part of the wave function. Symmetry in complex domain is an artifact and must not be mistaken for reality. Hermann Weyl at least did not mention this obligation when he confessed in [14]:

> *The problem of the proton and the electron is discussed in connection with the symmetry properties of the quantum laws with respect to the interchange of right and left, past and future, and positive and negative electricity. At present no acceptable solution is in sight’.*

Except for an anticipatory closed system, a future half cannot be derived from calculation. This substantiates the suspicion that the putative symmetry of the microscopic world is just a misinterpretation of the formal mirror-symmetry due to the arbitrary redundancy germane to complex Fourier transform. Why did nobody object to the grotesque idea of temporal mirror-symmetry in reality? Lawrence S. Schulman [15] even wrote:

> *Where is the frontier of physics? Some would say 10^{-33} cm, some 10^{-15} cm and some 10^{+28} cm. My vote is for 10^{-6} cm. Two of the greatest puzzles of our age have their*

origins at this interface between the macroscopic and the microscopic worlds. The older mystery is the thermodynamic arrow of time, the way that (mostly) time-symmetric microscopic laws acquire a manifest asymmetry at larger scales. And then there is the superposition principle of quantum mechanics, a profound revolution of the twentieth century. When this principle is extrapolated to macroscopic scales, its predictions seem wildly at odds with ordinary experience.

Physicists like Schulman understand so-called time-symmetry literally and not just like a metaphor paraphrasing the indifference of the laws of physics against shift of time and inversion of its direction. This is documented in papers like [15] and [16] where boundary conditions are symmetrically located in both the past and the future.

An author of a textbook on signal processing [17] referred to quantum mechanics when he claimed absolute symmetry with respect to t=0 in the nonsensical meaning that the future mirrors the past.

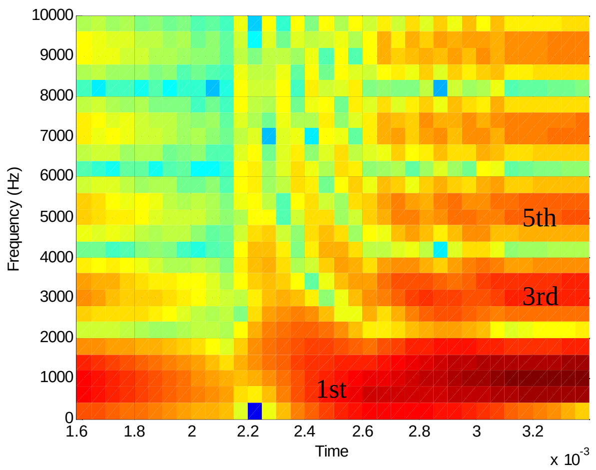

Fig. 3. *Matlab spectrogram for same f(t) as in Fig. 4*

Apparent symmetry also affects usual spectrograms, in particular narrowband spectrograms, most strikingly. They appear absurd in that they show values of frequency prior to the belonging step in the analyzed function of time.

Fig. 3 is an exception. Here the represented range of time is considerably cropped due to an extremely high degree of overlap (MATLAB 6.2 parameter Noverlap=63=Window-1). Therefore the usual non-causality is not obvious here. No advanced response to the left (last) steps is to be seen. Overlap was chosen as high as possible in order to reach for the first time at least similarity with the result shown in Fig. 4.

4. Cosine-transform (CT) pairs

Cosine transform does not yield the usual magnitude-style spectrogram but a time-frequency representation resembling the frequency analysis in cochlea (Fig. 4). It is based on exclusively positive elapsed time and accordingly positive frequency. It avoids the usual arbitrary choice of a reference and resulting redundancy. Therefore, it can be made accurate without restriction except for the limited sample rate while the usual spectrogram seriously suffers from trade-off between better time resolution but poorer frequency resolution or vice versa. Both the method and the results are much more natural with naked CT instead of FT. For instance, the ear does not know the moment of Christ's birth. A complex motion of basilar membrane in terms of magnitude, as shown in Fig. 3, could not at all be rectified while the alternating amplitudes with cosine spectrogram even allows one-way rectification differently depending on the sign of stimulus.

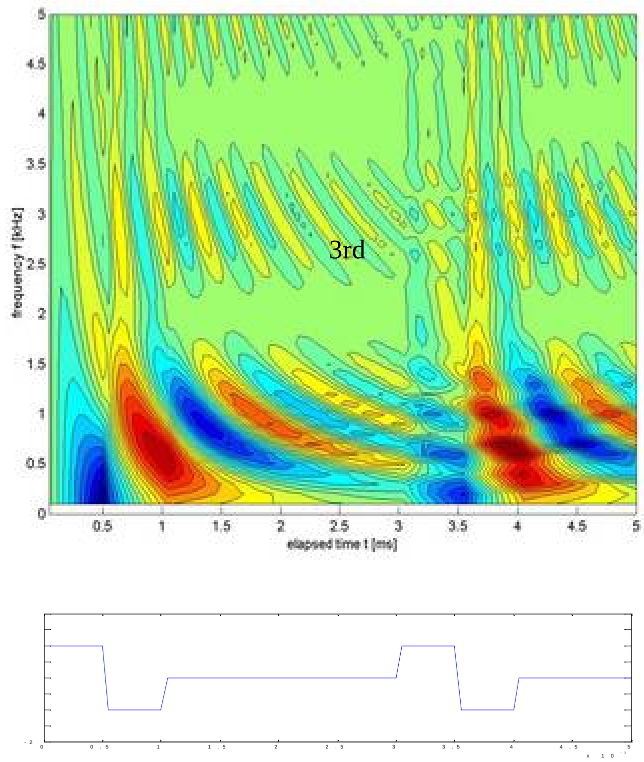

Fig. 4. *Cochleagram-style spectrogram based on CTs. Amplitudes alternate between negative and positive peaks in response to the steps at 4/3.5/3, and 1/0.5/0 ms before the now at the left margin. Imperfections due to the assumed sample rate. Below: steps of the stimulus.*

Fig. 4 does not yet consider the level-depending build-up of cochlear amplification. Apart from that it is in good agreement with physiological data from cochlea and auditory nerve[2]. It is the only model so far that convincingly explains how ears may distinguish rarefaction clicks from condensation clicks. All harmonics are odd. Because the analyzed function of time in Fig.4 consists of discrete steps, the corresponding elementary functions of frequency are continuous. Usual spectrograms are based on a short-time FT over arbitrarily chosen and repetitiously relocated windows of time. The cosine spectrogram is based on a natural time window, gradually set by the fading of amplitudes with

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[2] They show that traveling waves on basilar membrane are may be a phenomenon of active local resonance rather than the unrealistic energy transfer from base to apex, calculated by James Lighthill [18].

elapsing time. Since the cosine spectrogram opens the possibility to omit all redundant information, it may help to check interpretations that are meanwhile more or less accepted among physicists but still contradicting to common sense. Some experts did not trust in Fig. 4. When they argued that it did not allow for the uncertainty principle they denied the excellence of their own ears, too.

Fig. 5 may help to discuss their fallacy. It generalizes the first hyperbola in Fig. 3 and the first ripple ridge in Fig. 4. For the least discrete frequency and the also discrete minimal span of elapsed time Δt, the product is constant. The same applies for the product of the least discrete momentum pattern and the smallest distance of a wave or particle, respectively. CT has a property in common with FT. It changes discrete functions into continuous ones and vice versa. In that respect it does not matter whether the used kernel is a cosine or a complex exponential function. One may imagine a continuous function as an infinite amount of superimposed discrete ones and vice versa. The axes in Fig. 5 are dual to each other.

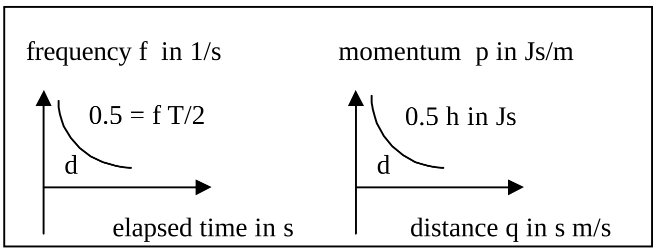

Fig. 5 *Simplified hyperbolas of least discrete products, d = area of alleged 'deafness'*

Duality is a general principle that concerns a huge diversity of mutually dual pairs. It does not just include pairs of physical quantities like energy or frequency matching to time, momentum matching to position, and radius to wave number. Electrical quantities correspond to magnetic ones. This means symmetry for related pairs like voltage and current, impedance and admittance, capacity and inductivity, in series and parallel, mesh and node. Duality is similar to orthogonality in that it unites mutually excluding but complementing components. Just some trifles like a different sign in Maxwell's equation disturb the symmetry. Though the electric field is intrinsically linked with the magnetic field, missing monopoles seem to indicate that the electric field precedes the magnetic one as do the current into a capacitor or the voltage at an induction coil. Similarly, elapsed time and distance might be primary with respect to frequency and momentum, respectively. These conjectured priorities are bound to the conditions that elapsed time is always positive, and only integration is realistic.

Most of the manifold dualities in terminology somehow relate to that relationship between electricity and magnetism or its mathematical basis: Wave vs. particle, infinity vs. zero, continuous vs. line spectrum, Heisenberg's vs. Schroedinger's picture, horizontal vs. vertical, range vs. resolution, ket vs. bra sine vs. cosine, and geometrical vs. wave optics/acoustics, respectively. Most general notions like rational, intensive, absolute, open, local, and finite have dual counterparts, too: irrational, extensive, relative, closed, global, and infinite. Some people even claim dualism between physical and mental properties. A typical dual pair consists of two distinct parts that can mutually exchange their role. Both are symmetrical in that they exclude but also complement each other. Awareness of dualities has many practical applications [19]. Set theory and the three spatial coordinates failed the duality check. Distance and elapsed time might constitute one dual pair. The two angles of sphere would then make up the remaining one.

While real part and imaginary part may favorably represent an orthogonal pair in phase domain, this is just one option. Dual physical quantities must not necessarily be considered this way. Because complex numbers do not at all exist in $\mathsf{R}+$, one cannot expect an imaginary commutator to distinguish there between commuting and non-commuting variables. Any function of solely positive elapsed time corresponds to the CT of a function of only positive frequency and vice versa. Together they constitute a CT pair of likewise real-valued quantities instead of the mixed FT pair, incorporating a real part and its twice-unphysical Hermitian counterpart.

A FT pair is known as two conjugate variables. Neither this nor Heisenberg's commutation conditions nor the so-called Pontryagin duality explain the mathematical reason behind uncertainty with a FT as well as a CT pair of the same two physical quantities. It is simply the crossing from finite to infinite or back, respectively, that makes FT pair and CT pair an uncertainty pair. The upper row in Fig. 2 reminds us that the cosine function has exactly the value 1 for a not quantifiable amount of arguments, and the opposite holds with its counterpart, the ideal impulse, which understood as an envelope, has no quantifiable value at a precise argument.

The cosine spectrogram of Fig. 4 represents an oscillating 3d-pattern on a membrane being undoubtedly continuous with respect to time, frequency and intensity. The allegedly unavoidable trade-off between resolutions in terms of frequency bandwidth and Δt has been completely removed because no arbitrary time window was used, and there was no obligation to restrict to discrete values of both variables. If the same could be reached for Fig. 3 by means of an infinite number of overlap, the result would still be different in that it did not show oscillations between positive and negative extremes but immediately smooth hyperbolically shaped magnitude ridges. In Fig. 4, the ridges for any particular contained frequency, are only imagined Hilbert envelopes. It is obvious that the two parts of any CT pair cannot be both discrete at a time. For that reason, exact measurement of corresponding discrete values is absolutely impossible. This is no peculiarity of quantum mechanics but holds true for all physics. Is there anything different with momentum and position? The least action principle by Pierre Maupertius (1698-1759) and Max Planck's quantum of action by were well known to the fathers of quantum mechanics. If there is no reason for time-frequency leaps, is there really a serious mathematical reason for postulating quantum leaps? When Nils Bohr continued to emphasize discrete energy states, he stressed the fact that neither the instant of decay nor the direction of emission is predictable from wave function. The Bohr school required that quantities should only appear in the theory if they were in principle measurable by spectroscopy. Accordingly, energy-related eigenvalues are real, but matrices depending on time or location are Hermitian.

In practical on-line analysis of continuously incoming signals, the area of alleged ‘deafness’ could be believed to not contribute anything. This would partly justify the lecturing: “Do not try to outwit Heisenberg’s uncertainty principle” [17]. However this advice is wrong because the ear is not obliged to measure discrete values.

Fig. 6 illustrates three options for the structure of the patterns in Figs. 3, 4, 5. Obviously, it depends on the sign of the first amplitude whether the first enveloping hyperbola is reached at one quarter or three quarters of a period. Net cochlear latency without delay due to resonance build-up in case of weak signals is half a period larger for positive as compared to negative clicks. The first realistic row of Fig. 2 is the middle one. Therefore, intensity does not start cosine-like. Operating with squared amplitudes, quantum mechanics neither benefits from the smoothness of the magnitude picture (Fig. 3) nor from the possibility to distinguish between positive and negative first ridges (Fig. 4).

Initially Heisenberg himself did not use the expression “uncertain” but “less precise” when he wrote in [20]:

> *The more precisely the position is determined, the less precisely the momentum is known in this instant.*

However, in the same paper he added a speculation:

I believe that the existence of the classical “path” can be formulated as follows: The path comes into existence only if we observe it.

This time Einstein defended realism. His primary concern was still his belief in key aspects of nature being independent of observers or their observations. He was reluctant to swallow Bohr’s ideas on complementarity including Max Born’s statistical approach, and he was perhaps quite right in so far as one must not attribute the uncertainty to the disturbance of the system under consideration by the measurement. The famous article [21] gave this telltale criterion of Einstein’s notion of physical reality:

> *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of reality corresponding to that quantity.*

Was not Einstein quite right when he derided entanglement as a spooky action at a distance? At least it looks surprising that

Schroedinger in his cat paper [22] quoted [21] as having suggested entanglement. This paper did not refer to two entangled particles but to

> *two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other”.*

The cat metaphor in [22] unnecessarily dramatized Einstein’s gunpowder argument that complained about the lacking intermediary: Both men agreed:

> *The psi-function must not describe a sort of blend of not-yet exploded and already exploded systems.*

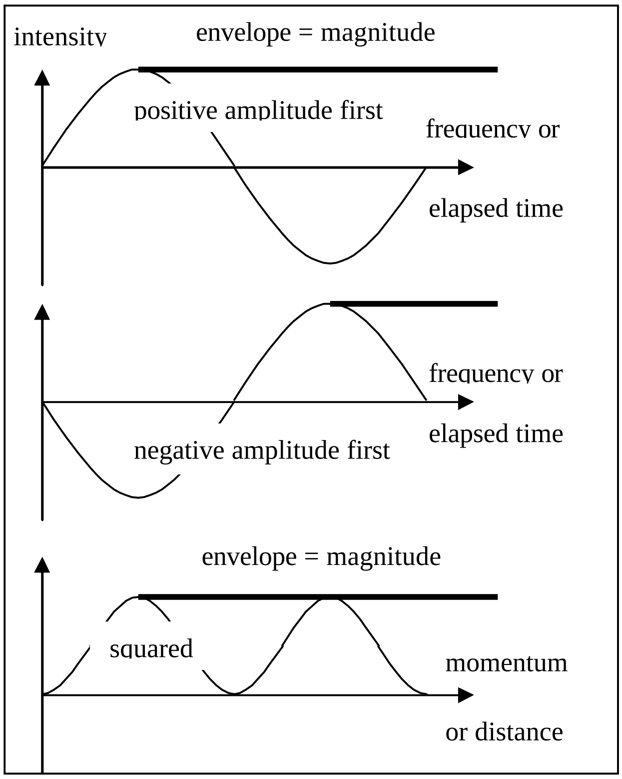

Fig. 6 *Simplified sections in Fig. 5. The two “fermionic” possibilities positive amplitude first($\pi/2$) and positive amplitude first ($3\pi/2$) exclude each other. The usually considered Schroedinger’s case $\psi\psi^*$ is shown below.*

Why did Schroedinger immediately take over the idea of an incomplete wave function? Already in 1926, Schroedinger wrote at the end (p. 139) of [13]:

> *If the use of a complex wave function was in principle*[3] *inevitable and not just a mere advantage in calculation, then this would imply that there are in principle two wave functions which only together give information about the state of the system.*

Schroedinger might have envisioned what is readable from Fig. 6 of this paper. Otherwise he was not quite correct. In case of the FT for a realistic physical quantity, i.e., a one-sided real function of either elapsed time or frequency, one needs both real and imaginary part in order to correctly represent just a single original function. It is seemingly obvious but fallacious that the two complex components represent a higher degree of freedom. As indicated above and

[3] While the proper meaning of the German word “prinzipiell” does not exclude exceptions, physicists like Einstein and Schroedinger rather meant “absolutely”

shown more in detail in [5], one single original function is four times represented except for sign matters.

5. Meta-mathematical background and implications

As is known, Einstein uttered disapproval of Hilbert's attack on Luitzen Brouwer. He nonetheless shared his view and called the controversy on fundamentals of mathematics pointless. Despite of being well guided by a sound common sense, Einstein failed to get aware of an admittedly deeply hidden mathematical deficit. Discrete values of a physical quantity necessarily reflect continuous functions of their CT (and FT) counterparts. When Einstein questioned the completeness of the quantum-mechanical description of physical reality, then he meant discrete values. He did not directly refer to the mathematical meaning of completeness, which was introduced by Richard Dedekind [23] in order force irrational numbers into the body of real numbers. Who was responsible? Hilbert was already retired. Among the liable mathematicians were his immediate successor Weyl and his pupil Johann v. Neumann. Weyl again confessed being "*less certain than ever about the ultimate foundations*", and the creator of Hilbert-space actually admitted in a letter to Garrett Birkhoff, dated Nov. 13, 1935, as a reaction to [20] and [21]:

> *I would like to make a confession, which may seem immoral. I do not believe absolutely in Hilbert-space any more. After all, Hilbert-space (as far as quantum mechanical things are concerned) was obtained by generalizing Euclidean space, footing on the principle of conserving the validity of all formal rules. This is very clear if you consider the axiomatic geometric definition of Hilbert-space where one simply takes Weyl's axiom for a unitary Euclidean space, drops the condition on the existence of a finite linear basis and replaced it by a minimum of topological assumptions (completeness + separability). Thus Hilbert-space is the straightforward generalization of Euclidean space if one considers the vectors as the essential notions. Now we begin to believe that it is not the vectors which matter but the lattice of all linear (closed) subspaces.*

Key words are completeness, separability, and generalization. Ideal functions that range from -oo to +oo including sine cosine and exponential functions are as unrealistic as is the ideal impulse: Future values are definitely missing and amplitudes are never infinitely large. The problem turns out to be a purely meta-mathematical one. Hilbert space cannot furnish discrete values for both parts of a CT or FT pair simultaneously, because unrestricted completeness up to infinity is required when performing the integration with cosine or exponential kernel. The same problem faced the general definition of any set including infinite ones given by Cantor [24], p. 3:

> *A set is a combination of certain well-distinguished objects of our experience or our thinking – which are called elements of the set – into an entity.*

This definition led into paradoxes and was therefore declared invalid. Adolf Fraenkel [24], p.185, wrote:

> *Any definition of the notion "set" and of the relation "m is element of the set M" will be disregarded at all; the [above quoted] definition by Cantor has proven untenable due to the paradoxes - i.e., eventually the method to ascribe to any logical notion a set that denotes its size has been abandoned and not replaced by any new definition of set.*

The abandoned definition of an infinite set suffered from the same illusion, as does the already quoted claim of mutually precluding knowledge of non-commuting operators [21].

One cannot have mutually excluding points of view on the same object simultaneously. In case of an infinite set, one has to choose between two views, either the element-by-element Archimedean one, attributing a successor to any number or, at higher level of abstraction, the fictitious entity of all numbers. It is impossible to resolve all elements out of an absolutely infinite entity. Exp(x) cannot be quantified for an indefinitely large argument x. The other way round, one cannot attribute a number to exp(x)=0.

Schroedinger's idea of not just entangled state vectors but also entangled particles and belonging decoherence even led to the idea of quantum computing. Should one really comment on this with the same sibylline quotation that Hans-Dieter Ebbinghaus [25] put below the title of his chapter on Set Theory and Mathematics without revealing the fallacy? What is true and what is great on it?

> *Given, there was a great useful mathematical truth, the inventor of which was guided to by an obvious fallacy; - and if there is nothing of this kind, it nonetheless could exist – do I therefore deny this truth, do I renounce the opportunity of using it?(LESSING, Theol. pamphlets)*

Questions of this kind go beyond the scope of this paper. If genuine success in research on quantum computing remains questionable, then serious doubts deserve due consideration.

6. Conclusion: Generalization only on demand

Consequent obedience to the still valid argument by Ritz gave rise for putting in question the customary belief in that maximal generalization is always the best and only option in science. Science undoubtedly benefits a lot from the usual notion of time, general solutions of DEQs, and complex calculus. On the other hand, restrictive adaptation to what makes the physical reality special avoids the need for enforcing generality without any benefit. Suitable adaptation means less general mathematical tools.

There is nothing wrong with arbitrarily chosen redundant data on condition one correctly interprets the results and rejects sentences like "the solution is in general complex" as thoughtless and ambiguous. Removal of redundancy would remove obstacles that have been hampering the understanding of not yet convincing tenets. In particular, the argument by Ritz may hopefully contribute to a purification of quantum mechanics from rather spooky and scurrile counterintuitive "facts". The final word will have the pertaining experts. They are invited to check the argument by Ritz again. In summa, it seems as if quantum mechanics is not as special as it is regarded to be.

- Alleged T-symmetry in the sense of mirror symmetry only in the microscopic world has molten down to the ubiquitous ambivalence of general solutions to DEQs.

- Uncertainty is just a meta-mathematical aspect of CT pairs and not at all restricted to quantum mechanics. Any CT pair consists of two likewise physical real parts mutually complementing each other with respect to discreteness or continuity.

- Pauli and others [26] claimed that quantum mechanics is the first discipline that could under no circumstances renounce the imaginary unit. However, nobody was able to prove this guess. Complex calculus has generally proven a clever tool for two mutually related quantities while it is an avoidable source of confusion in analysis of just one quantity.

- Is there any compelling reason for interpreting wave functions in terms of probability? Nobody would like to do so with Fig. 4. When Born and Bohr argued for an interpretation of Ψ as probability of location for a point, they did not consider that ideal points are unreal while the naturally localizing functions sinc and sinc^2 remain finite.

While the odd symmetry of Ψ for fermions cannot be directly expressed within $\mathbb{R}+$, it could be assigned to a positive as well as negative sign of 'spin'. Bosons can be circularly polarized. Isn't it a tricky question whether a single photon is linearly or circularly polarized since a single circularly polarized electron could naively be thought to equal two linearly polarized ones or also naively the other way round? Equal treatment requires considering any 'single' electron twinned with itself. May we guess that bosons and fermions are likewise physically real if considered each from its own perspective?

Be not worried by the authors naïve guesses in the area of particle physics. He managed to show not just the equivalence between the CT and the FT of unilateral functions. He also succeeded in changing the style of spectrograms from cochleagram to the usual magnitude style by means of Hilbert transform.

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