

The relativistic thermodynamics

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A very narrow issue of relativistic thermodynamics of a closed system was addressed in the article. Problem analysis in terms of timeless model was made.

Following a creation of a special relativity theory relativistic transformations for temperature and quantity of heat were obtained. We will make their overview from H. Callen and G. Horwitz article [1].

Thermodynamics was adapted to new covariation requirements by Plank [2] and Einstein in 1907 [3]; it seemed that relativistic thermodynamics reached a reasonable completion. Temperature and heat flow (quantity of heat transferred to a system) transformation laws were its basic results. It stemmed from Plank work that the system will be colder in terms of an observer moving relative to it and that heat flow will be respectively lower:

$$T = T^{(0)} \left[1 - (v^2 / c^2) \right]^{1/2} \equiv T^{(0)} / \gamma, \quad \Delta Q = \Delta Q^{(0)} / \gamma \quad (\text{Plank}). \quad (1)$$

In 1963 Ott [4] once again returned to this issue and insisted that he managed to come to transformation laws by means of a completely reverse way than that of Plank. According to Ott, a moving observer finds that the system is hotter and heat flow - bigger:

$$T = \gamma T^{(0)}, \quad \Delta Q = \gamma \Delta Q^{(0)} \quad (\text{Ott}).$$

Afterwards, other works followed. Landsberg [5] assumed that temperature should be considered to be a Lorentz invariant whereas an old Plank transformation for transferred heat remains true:

$$T = T^{(0)}, \quad \Delta Q = \Delta Q^{(0)} / \gamma \quad (\text{Landsberg}).$$

Some time later Van Kampen [6] proposed a scheme with invariable temperature and invariable heat flow:

$$T = T^{(0)}, \quad \Delta Q = \Delta Q^{(0)} \quad (\text{Van Kampen}).$$

However, even this list doesn't exhaust all opportunities; not only other proposals with respect to T and ΔQ transformation laws exist but also various conclusions of the above ratios based on different definitions of other thermodynamic values, for example, such as pressure, operation, enthalpy. Existence of various opportunities, each of which is allowable, puts solution of issue on construction of a relativistic thermodynamics in a very unpleasant position [1].

According to the first law of thermodynamics, total system energy which we will indicate as E is a single-value function of thermodynamic system condition. If a system (body) takes part in a process which causes variation of body condition, then energy variation is defined by means of formula:

$$\Delta E = \Delta Q + \Delta A \quad (2)$$

where ΔQ - quantity of heat transferred to system within a process and ΔA - mechanical work completed with the system by bodies surrounding it. Certainly, work of mechanical forces can include work of electromagnetic field forces if such forces affect the system. The first law represents energy conservation law, and its general view should be retained in all inertial reference systems. Thus, an equation (2) must be true in any inertial system.

We shall note that transition from one inertial reference system to another is largely defined by the fact that transformation formulae for ΔE and ΔA result from relativistic mechanics of continuous media; thus the issue is limited to a transformation definition for ΔQ . The problem is that in relativistic thermodynamics it is difficult to separate the work of external forces for a mechanical impulse and heat transfer. Relativity of time and anisotropism of volume don't enable to consider impulse of substance inertia centre and temperature as independent parameters.

Let us make only a brief conclusion of Plank and Ott solution and restrict ourselves to analysis of differences. For more detailed information see represented literature and numerous reviews.

Let's calculate work completed with a system, and we shall use a term of work only in a limited sense applied by classical mechanics. In a native system work is equal to multiplication of pressure by $PdV^{(0)}$ volume variation. In order to calculate work in a moving system, first of all, we shall write down energy in a moving system, namely:

$$E + PV = \gamma (E^{(0)} + PV^{(0)}) \text{ or } E = \gamma E^{(0)} + \gamma (v^2 / c^2) PV^{(0)},$$

$$dE = \gamma dE^{(0)} + \gamma (v^2 / c^2) P dV^{(0)} + \gamma^2 (v^2 / c^2) V^{(0)} dP =$$

$$= \gamma^{-1} P dV^{(0)} + \gamma (v^2 / c^2) V^{(0)} dP, \quad dE = -P dV + \gamma^2 (v^2 / c^2) V dP \quad (3)$$

It should be emphasized that in a native system a purely mechanical process of thermodynamic work $dE^{(0)} = -P dV^{(0)}$ is considered which stems from an effect of mechanical force $dE^{(0)} = F d\ell$. From a point of view of an outside observer, we find that mechanical work is described by means of both a member containing dP and a familiar member containing dV . This result plays an essential role in all controversial assumptions. It is suitable for an ideal gas as well which is limited by walls and for a liquid substance.

It is possible to show that subsequent relativistic equations of thermodynamics can be obtained by means of two ways. A fundamental ratio for enthalpy retains its relativistic form, but enthalpy must be interpreted according to the following ratio (4).

In a native reference system this dependant variable is equal to

$H = E^{(0)} + PV^{(0)}$ and represents a zero component of 4-vector of moment of impulse $H^{(0)} = \{c^{-1}(E + PV), p_x, p_y, p_z\}$ so that in a native system it would be limited to $E^{(0)} + PV^{(0)}$, then H should be defined through Lorenz-invariant absolute value H :

$$H \equiv c|H| = \left[(E + PV)^2 - c^2 p^2 \right]^{1/2} \quad (4)$$

All the rest ratios of thermodynamics can be obtained by applying common transformation techniques. By means of another way it is possible to start from any fundamental equation of non-relativistic thermodynamics in a native reference system by transforming each variable separately to a moving system. By doing so, all intensive parameters (T, P, μ) shall be considered to be Lorenz-invariant ones whereas free energies must be transformed in accordance with transformations of enthalpy. Variations with other presentation of thermodynamic characteristics don't justify results ¹ because heat and temperature are not covariant, and their values cannot be obtained directly from Lorenz transformations.

1. History of relativistic thermodynamics is detailed brilliantly by Smid [7], [8] (who also used definition of temperature by means of 4-vector when he was engaged in a research of relativistic liquids).

Heat transfer for closed systems is defined by means of

$$\Delta Q = \Delta E - \Delta A, \quad E + PV = \gamma(E^{(0)} + PV^{(0)}) \text{ if } dN = 0. \quad (5)$$

thus,

$$\begin{aligned} dE &= \gamma dE^{(0)} + \gamma(v^2/c^2)P dV^{(0)} + \gamma(v^2/c^2)V^{(0)} dP + (E^{(0)} + PV^{(0)})d\gamma = \\ &= \gamma(T dS - P dV^{(0)} + \mu dN) + \gamma(v^2/c^2)P dV^{(0)} + \gamma(v^2/c^2)V^{(0)} dP + \\ &+ \gamma^3(E^{(0)} + PV^{(0)})v(dv/c^2), \end{aligned} \quad (6)$$

The last member of (6) can be simplified by remembering

$$p = \gamma(v^2/c^2)(E^{(0)} + PV^{(0)}) \text{ transformation.}$$

Thus

$$dE = \gamma T dS - P dV + \gamma^2(v^2/c^2)V dP + \gamma^2 p dV + \mu dN, \quad (7)$$

where the first member represents a variation of entropy flow, and the second and third members represent a mechanical work which we already met in formula (3); the fourth member represents an additional expression which should be included in ΔA . It takes into account work on acceleration of centre of system inertia. The fifth member is a variation of static condition of thermodynamic system.

Where it stems from (7)

$$dQ = \gamma T dS = \gamma dQ^{(0)}. \quad \text{This is the result of Ott.}$$

The result of Plank stems from formula (4) by introducing a relative enthalpy $\tilde{H} \equiv E + PV$, it should be distinguished from H invariant enthalpy.

$$\tilde{H} = (H^2 + c^2 p^2)^{1/2}$$

From Lorenz transformations it stems that $H/\tilde{H} = \gamma^{-1}$ and that $pc^2/\tilde{H} = V$, then $\tilde{H} = [T/\gamma]S + [\mu/\gamma]N + [v]p$, here intensive parameters are taken in brackets. Omitting intermediate transformations and assuming that

$$\tilde{H} - PV = E \quad \text{we obtain} \quad dE = [T/\gamma]dS - [P]dV + [\mu/\gamma]dN + [v]dp \quad (8)$$

This implies the result of Plank $dQ = [T/\gamma]dS$.

Why is it incorrect just to put $dS = dN = 0$ in identical equation (8) and consider the obtained expression to be a work? This is a common procedure by means of which an expression for work in common thermodynamics is obtained. It is based on an idea that work is defined by energy transfer which occurs in case of constant entropy and constant number of moles. Failure of this idea is reflected by an obvious fact that p impulse defines not only movement of centre of inertia. In a total impulse internal and external attributes are mixed; flow of heat inside a system occurring with a constant speed alters p .

What is the meaning of Plank and Ott difference?

They interpret work, which is completed by an external force in a different way.

Plank defined force as time derivative from $\frac{dp}{dt} = F_s$ impulse, thus, in a moving reference

system νdp work is completed with a body. Ott assumed that purely mechanical part $dp_M = (E + PV)d\nu$ and "heat" part $dp_T = \nu d(E + PV)$ should be separated from a total impulse; besides, he referred work completion only to a mechanical part of impulse variation, and "heat" part describes work just as a part of transferred heat.

We shall write down a total working force in an explicit way:

$$\frac{dp}{dt} = F_M + F_T = \frac{d}{dt} \left(\frac{m_0 \nu}{\sqrt{1 - \beta^2}} \right) = m_0 \frac{d}{dt} \frac{\nu}{\sqrt{1 - \beta^2}} + \frac{m_0 \nu}{\sqrt{1 - \beta^2}} \quad (9)$$

The first member of sum describes system movement with acceleration and contains t time parameter; the second member of sum describes inertial system movement. It is understood as a non-mechanical variation of impulse which stems from rest mass variation or as a relativistic self-deformation of a substance. If in unit time Q heat quantity is given to a body, then a body acquires $Q\nu/c^2$ impulse.

It is assumed that a relativistic decrease of molar volume of substance is not accompanied by surmounting any forces of resistance to it. And, thus, this decrease is an inertial self-deformation of a substance which is strictly agreed with variation of its movement speed. Because a rigid body resistant to deformation in statics (of whatever big force) is susceptible to relativistic deformation under the effect of a substantially smaller force which accelerated its movement. Thus, a relativistic self-reduction of molecular volume of a substance is not an effect of any force. It results from adaptation of elementary particles, atoms and molecules of this substance to changed conditions of their interaction [9].

Thus, a relativistic reduction of body length and molar volume of its substance is not considered in a special relativity system as a purely kinematic effect which is not accompanied by energy input for the performance of work on relativistic deformation of a substance. And if this energy was consumed not only for increasing impulse of a substance but also for its relativistic self-compression, then the following effect would occur. In the course of strictly dynamic consideration of movement which doesn't take into account these additional energy inputs energy conservation law wouldn't be fulfilled in mechanics. Thus, probably there is no need for taking into account the second member of sum in expression (9)? No, it is necessary particularly for observing energy conservation law in thermodynamics. Both Plank and Ott took it into account (everybody by means of its own way).

As it is well-known, relativistic thermodynamics is based on an assumption that an object (substance) can be sped up to any speed with infinitely small acceleration. Such a process will be adiabatic. i.e. it won't touch thermodynamic condition of an object. On the other hand, it is suggested that a number of microconditions of a system cannot depend on a condition of movement of outside observer. Let's consider these provisions in terms of space-and-time quantization.

Let's assume that every quantum transition of microparticle condition alters its local space and moves its local time by one point forward. This location is confirmed by a definition of a wave function: condition of microparticle remains constant until reduction of a wave function takes place².

Let's redefine temperature as a combination of intensity of quantum transitions.

Generally energy spectrum of system from N molecules is represented in a form of two parts: 1) spectrum (usually discrete) stemmed from quantum transitions in individual molecules is a task of quantum mechanics for this configuration of atoms and 2) spectrum stemmed from heat, progressive motion of particles. For an ideal gas it is primary. Wave function with N particles condition in $V = L^3$ volume:

$$\psi_n(x) = C \sin k_n x, \quad k_n = \frac{\pi n}{L}, \quad n=1, 2, 3, \dots$$

then energy spectrum

$$E_n = \frac{(\hbar k_n)^2}{2m} = \frac{\hbar^2 \pi^2}{2m} \frac{n^2}{L^2} \text{ in case when } N \rightarrow \infty \text{ spectrum is asymptotically continuous.}$$

Let's apply microcanonic distribution of Gibbs [10].

Δ - function

$$w_n(\varepsilon, x, N) = \left\{ \begin{array}{l} \frac{1}{\Gamma(\varepsilon, x, N)}, \text{ when } E_n < \delta E \\ 0, \text{ when } E_n > \delta E \end{array} \right\} = \frac{\Delta(\varepsilon - E_n)}{\Gamma(\varepsilon, x, N)}$$

We shall note that in this equation summing occurs not by levels of E_n energy but by $\varepsilon - E_n$ all various microscopic conditions of a system in question described by means of wave functions $\psi_n(\varepsilon, x, N)$.

2. As an illustration, local space can be represented as a pattern, an analogue of which can be a relativistic electron accelerating in electromagnetic field. With each quantum transition the pattern changes: photon radiation occurs mainly in a direction ahead of itself. With each quantum transition radiation cone becomes narrower.

Statistical weight of thermodynamic equilibrium, i.e. number of microscopic implementations of this macrocondition (ε, x, N)

$$\Gamma(\varepsilon, x, N) = \sum_n \Delta(\varepsilon - E_n(x, N)).$$

Γ - quantity is discrete, in fact it is a parameter of time t , a counter of quantum transitions, but to use it very difficultly. We shall note that $N \rightarrow \infty$, $\Gamma \rightarrow \infty$.

This quantity is more interesting $\frac{d\Gamma}{\Gamma} = d \ln \Gamma$, and entropy $S(\varepsilon, x, N) = \ln \Gamma(\varepsilon, x, N)$

Then entropy variation $dS = \frac{\partial \ln \Gamma}{\partial \varepsilon} (d\varepsilon + X dx - \mu dN) = \frac{\partial \ln \Gamma}{\partial \varepsilon} \delta Q$, where

X - thermodynamic force associated with variation of x parameter,
 μ - chemical potential, this is a variation of internal ε energy in the course of adding one particle to a system, provided a system doesn't complete work and doesn't receive heat,

dN - variation of molecules' number or system microconditions (for example, in case of phase transition).

Then absolute system temperature

$$\frac{1}{T} = \frac{\partial \ln \Gamma}{\partial \varepsilon} = \frac{\partial S(\varepsilon, x, N)}{\partial \varepsilon}.$$

Pay attention, that in a condition of thermodynamic rest $T - const$, $S - const$, $\frac{d\Gamma}{\Gamma} \approx const$,

but Γ - increases. An environment maintains thermodynamic rest.

Thus, temperature can be interpreted not only as a parameter of energy state of equilibrium system but also as a parameter of its energy state variation and as a combination of intensity of quantum transitions.

Let's turn once again to expression (9). The first member of sum describes movement with acceleration. If one considers substance as a combination of local spaces, then work on acceleration is related to quantum reforming of local space. Every quantum transition reduces anisotropy of local space and makes "tick" to time. Consequently, thermodynamic system cannot be accelerated adiabatically.

The second member of sum describes inertial movement. Quantum transitions related to reforming of space are not suitable there. Static local space implies a constant temperature and lack of time gradient. But it takes into consideration relativity of movement. We are interested in time delay in an initial moment of thermodynamic process due to which an additional impulse arises. Two inertial reference systems move relative to each other already having a certain anisotropism of space. In order to consider this anisotropism a certain quantity of heat has to be

given to a system, i. e. this is a fictive work; it was already performed in the course of systems' acceleration. But it looks like an outside observer changes a number of microconditions of a system in question.

In our opinion, the problem is resolved by the fact that we must consider anisotropism of space from the outset. For this purpose, distribution of energy by degrees of freedom in a thermodynamic process has to be considered, depending on a relativistic speed.

For example, a universal R gas invariable is equal numerically to expansion work of one mole of ideal gas in isobaric process in case of temperature variation by one degree. It is required to define $E \equiv f(R, \nu)T$ function,

in the course of rate fixing of $E^{(0)}, T^{(0)}, \nu^{(0)}$, $f(R) = R^{(0)}$,

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