Scales Solve the Continuous vs. Discrete Paradox

by Ray B. Munroe, Jr.*

Abstract – It is the author's position that Nature is fundamentally both continuous and discrete, and that this paradox is directly responsible for the wave-particle duality of Nature. Two key catalysts in the collapse of continuous wave functions (waves) into sets of discrete quantum numbers (particles) are 1) Scales and 2) Lucas Numbers. These concepts – along with Supersymmetry – may provide the framework for the ultimate unification of bosons and fermions.

Introduction

Many classical physical properties seem to be continuous in nature, such as position, time, momentum and energy. Some physical properties, such as the rest mass of fundamental particles, seem to exhibit discrete, but non-quantized values, such as the electron rest mass of 510.9989 KeV/c^2. However, relativistic mass based on $E = mc^2 = m_0 c^2 \Big/ \sqrt{1-\left(\frac{v}{c}\right)^2}$ can take on seemingly continuous values (above a lower cutoff value of m_0 – the rest mass). In contrast, many quantum physical properties seem to be discrete in nature. For example, Table 1 may be the beginning of a discrete multi-dimensional lattice (except for the Weak Hypercharges of right-handed particles as the author has previously addressed [1]) – similar to Lisi's E8 "Exceptionally Simple Theory of Everything (TOE)" [2] that was based on the 8-dimensional Gosset lattice. Here, (g_3, g_8, Y_W, T_3) are the traditional Color, Weak Hypercharge, and Weak Isospin "charges" of the Standard Model [3], $SU(3)_C \times U(1)_Y \times SU(2)_W$:

Table 1 – Some Examples of the Discrete Nature of Fundamental Particles

Particle Name	Symbol	g_3 Color	$\sqrt{3}\, g_8$ Color	$-\frac{3}{2} Y_W$ Weak Hypercharge	T_3 Weak Isospin
Left (-handed) electron	e_L	0	0	$\frac{3}{2}$	$-\frac{1}{2}$
Left electron neutrino	υ_{eL}	0	0	$\frac{3}{2}$	$\frac{1}{2}$
Right (-handed) electron	e_R	0	0	3	0
Left red up quark	u_L^r	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$
Left green up quark	u_L^g	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$
Left blue up quark	u_L^b	0	-1	$-\frac{1}{2}$	$\frac{1}{2}$
Left down quark	d_L	Same (r,g,b) (red, green, blue) color options as above quarks		$-\frac{1}{2}$	$-\frac{1}{2}$
Right up quark	u_R			-2	0
Right down quark	d_R			1	0

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Physical and Mathematical Background to the Importance of Scales:

In covariant notation, the Dirac Equation [4] is:

$$-i\hbar\gamma^{\mu}\partial_{\mu}\psi + mc\psi = 0 \tag{1}$$

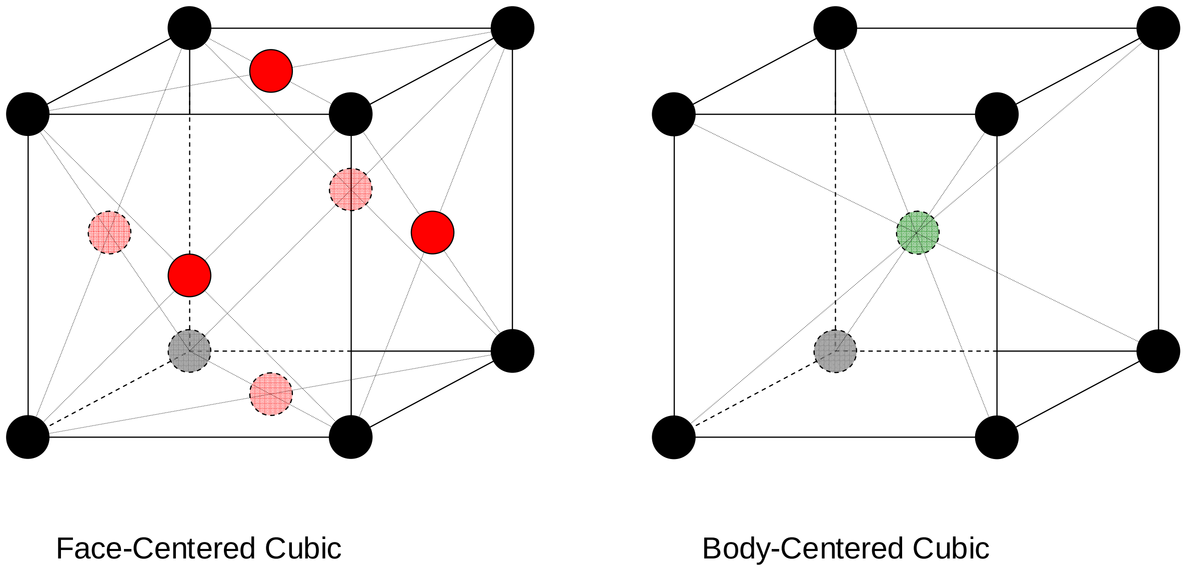
To solve this equation, Dirac assumed the Dirac Sea – that the vacuum (yes, the same vacuum of the "aether" or the Standard Model Higgs theory) contains an "infinite sea" of positive energy and negative energy states that are somewhat analogous to a solid state crystal containing an "infinite" number of electrons and holes. In the context of the Dirac Equation, positive energy solutions are normal matter such as electrons, and negative energy solutions are anti-matter such as positrons. Whenever we excite a particle out of the Dirac Sea, it is naturally paired with an anti-particle that flies off in the opposite direction (in the center-of-mass reference frame). In the context of solid state physics, a hole is a quasi-particle [5] lattice defect that behaves like a positive electric charge as opposed to an electron with negative electric charge or an atomic nucleus of different mass. In many ways, these physical effects – the Dirac Sea vs. quasi-particles in an "infinite" discrete solid state lattice – are analogous.

If the Dirac Sea (which represents the vacuum and the very "fabric" of Spacetime) behaves like a discrete multi-dimensional close-packing lattice, then we can benefit by studying the 3-dimensional Face-Centered-Cubic (FCC) close-packing lattice of solid state physics. The FCC unit cell is a cube with an eighth of an atom on each of the cube's eight corners and a half of an atom in the center of each of the cell's six faces (red in Figure 1), such that the FCC unit cell contains four atoms, and this lattice may extend to "infinity" in all three spatial dimensions by stacking cubic unit cells beside and on top of each other. In Solid State Physics, we also study the reciprocal lattice, where the matrix inversion form of this operation is [6] :

$$[\mathbf{b_1},\mathbf{b_2},\mathbf{b_3}]^T = 2\pi[\mathbf{a_1},\mathbf{a_2},\mathbf{a_3}]^{-1} \tag{2}$$

Where $(\mathbf{a_1},\mathbf{a_2},\mathbf{a_3})$ are direct primitive vectors each with *length* units, and $(\mathbf{b_1},\mathbf{b_2},\mathbf{b_3})$ are reciprocal primitive vectors each with *inverse length* units. These inverse lattices represent two self-similar scales [7] and the normalization factor for these two scales is 2π . The reciprocal lattice of an FCC lattice is a Body-Centered-Cubic (BCC) lattice. In contrast to the FCC unit cell, a BCC unit cell is a cube with an eighth of an atom on each of the cube's eight corners and a whole atom at the center of each of the cube (green in Figure 1), such that the BCC unit cell contains two atoms.

Figure 1 – FCC and BCC Lattices



Supersymmetry (SUSY) [8] introduces sparticles that are related to known particles via an assumed R-Parity. SUSY was introduced to solve the Standard Model Hierarchy Problem – "Why are both the Weak and TOE energy scales stable?" which IS a Scale Problem. In addition, SUSY may be a legitimate Particle Physics analogy of direct (Fermions) and reciprocal (Bosons) lattices. Within this interpretation, Lisi's [2] direct Gosset lattice vertices represent Fermions, and the struts (reciprocal lattice vectors) that connect these vertices represent Bosons. The difference is that SUSY requires an equal number of degrees-of-freedom (dgf's) from particles and sparticles, thus implying that we must double the number of BCC dgf's to match the number of FCC dgf's. This may be the origin of Fermions with spin $\pm\frac{\hbar}{2}$ – providing left- and right-handed helicities.

In Solid State Physics, the direct vectors describe position, whereas the reciprocal vectors describe momentum. As such, these direct and reciprocal vectors represent two different scales, and Heisenberg's Uncertainty Principle [9] may be an artifact of the union and normalization of these two scales:

$$\Delta x \Delta p \geq \frac{\hbar}{2} \tag{3}$$

The recognition of the necessity for two different scales (technically, a Classical and a Quantum) is important because the same concepts may apply to a close-packing Dirac Sea, but we may be unable to fully and accurately probe the Dirac Sea scale †. This reciprocal lattice transform can be extended to more "dimensions", and has the basic features of a multi-dimensional quantized Fourier transform:

$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i x \xi} d\xi \tag{4}$$

Is Mathematics Naturally Continuous or Discrete?

What is the most natural basis for mathematics? We are naturally biased to prefer numbers that we can count on our fingers – the discrete integers: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10…, and Quantum Theory utilizes these integers. However, Statistical Mechanics is based on more-or-less continuous exponential powers: $\exp(x)$ – and these exponential powers are much better at defining very large or very small scales, such as Dirac's Large Number of $\sim 10^{40} \sim \exp(92)$ [10] or the Cosmological Constant of $\sim 10^{-120} \sim \exp(-276)$. Can we find a mathematical "bridge" between these two extremes of continuous vs. discrete numbers?

The Importance of the Golden Ratio

The Golden Ratio [11] is given by the solution to a simple quadratic equation: $x^2 - x - 1 = 0$, such that $x = \frac{1 \pm \sqrt{5}}{2} = \left(\phi^1, -\phi^{-1}\right) = \left(1.618034..., -0.618034...\right)$. This ratio describes the relative chord lengths of pentagrams inscribed within pentagrams [see Appendix Figure A1.], and Fibonacci's sequence [12]: $(1,1,2,3,5,8,13,21,44,...)$ divided by five yields rough powers of the Golden Ratio: $(1,1,2,3,5,8,13,21,44,...)/5 \sim \left(\phi^{-4}, \phi^{-3}, \phi^{-2}, \phi^{-1}, \phi^{0}, \phi^{1}, \phi^{2}, \phi^{3}, \phi^{4}, ...\right)$. The Golden Ratio is also special because adding two successive powers of the Golden Ratios is equivalent to multiplying the larger power by ϕ: $\phi^{l+1} = \phi^{l} + \phi^{l-1}$. [see Appendix Table A1.] Thus, the Golden Ratio has simplified logarithmic properties ‡, and is "Nature's Slide Ruler" in the interpretation that multiplication (and powers of "continuous" exponentials) is simplified to addition. For example, $\phi^1 + \phi^2 = 1.618034... + 2.618034... = 4.236068... = \phi^3 = \phi^1 \times \phi^2$ mirrors logarithmic properties: $\ln(\phi^1) + \ln(\phi^2) = (1+2) \times \ln(\phi) = 3 \times \ln(\phi) = \ln(\phi^3)$.

† If we were able to fully explore the Dirac Sea scale, then we would better understand the origin of mass.

‡ The natural logarithm (ln) is the inverse of the exponential function (exp) such that $x = \exp(\ln[x])$ for $x > 0$.

A Mathematical Oddity that Unites Different Scales:

Lucas Numbers [13] have properties that allow us to transform irrational numbers into integers by adding powers of the Golden Ratio and its inverse: $\left(\phi^1, -\phi^{-1}\right) = \frac{1 \pm \sqrt{5}}{2} = \left(1.618034..., -0.618034...\right)$. This allows these Lucas numbers, $n_l = \phi^l + \left(-\phi\right)^{-l}$ and $n_{l+2} = n_l + n_{l+1}$ such that $n_l = \left(2, 1, 3, 4, 7, 11, 18, 29, 47, 76, \ ...\right)$ to unite inverse scales, such as $\phi^1 - \phi^{-1}$, $\phi^2 + \phi^{-2}$, $\phi^3 - \phi^{-3}$, etc. [see Appendix Table A2.] If these pairs of inverse scales represent our quantum-scaled direct lattice (Hyperspace?) and our classical-scaled reciprocal "lattice" (Spacetime?) [§], then we may use properties of Lucas Numbers to "twist" these scales together into twistor-like quasi-particles [14,5] and "quasi-dimensions". Now we understand why we may treat "light" as a classical-scaled continuous electromagnetic wave OR as a quantum-scaled discrete photon particle – because our concept of "light" is one of these quasi-particles created by intertwined scales. This explains the origin of Wave-Particle Duality with the observation that waves have continuous classical-scaled properties and particles have discrete quantum-scaled properties.

Another Potential Application of Lucas Numbers

If Scales and Supersymmetry (SUSY) are related via these direct and reciprocal lattices, then we should be able to simplify our spectrum of spin statistics (Maxwell-Boltzmann, Fermi-Dirac, and Bose-Einstein) in terms of Lucas numbers [15]. This also describes Nature in terms of a self-similarly-consistent discrete (Fermions with occupation values of zero or one) versus continuous (Bosons with occupation values of zero to infinity) Dual Nature. The continuous extreme of this spectrum represents wave-like Strings / Membranes, real numbers / "Infinity" and Bosons; whereas the discrete extreme represents particle-like Kissing Spheres [1,2] / Causal Dynamical Triangulation (CDT) [16], integers and Fermions. It is well-known that we can combine these three forms of spin statistics (Equation 5) [17] into one form (Equation 6 and Table 2), but this new insight into Scales allows us to represent our Θ term (degree of discreteness (-1) vs. continuity (+1)) in terms of Lucas numbers. Table 2 is just the beginning of the Lucas sequence, but these numbers represent the greatest extremes. Other effects can be corrected with ultraviolet or infrared corrections and phase space / microstate distributions.

$$f_{MB} = [\exp(\beta E_n)]^{-1}, \qquad f_{FD} = [\exp(\beta E_n) + 1]^{-1}, \qquad f_{BE} = [\exp(\beta E_n) - 1]^{-1} \tag{5}$$

or $$f_{\Theta} = [\exp(\beta E_n) - \Theta]^{-1} \quad \text{where } \Theta = (-1 \text{ Fermi, } 0 \text{ Maxwell, } +1 \text{ Bose}) \tag{6}$$

[§] Recognize that Spacetime does not exhibit discrete lattice-like properties because the number of states in the Classical Scale is so large that they seemingly blend into a continuum.

Table 2 – Lucas Numbers and Spin Statistics

l	Lucas Numbers: $n_l = \left(\phi^l + (-\phi)^{-l}\right)$	$\Theta = n_l - n_0 = n_l - 2$	Symmetry:	Spin Statistics:
0	$2 = 1.000... + 1.000...$	0	Identical	Maxwell-Boltzmann Continuous $[0, \infty)$
1	$1 = 1.618... - 0.618...$	-1	Odd Parity	Fermi-Dirac Discrete $[0,1]$
2	$3 = 2.618... + 0.382...$	$+1$	Even Parity	Bose-Einstein Continuous $[0, \infty)$

Speculations on the Relevance of Scales to a Theory of Everything (TOE)

Supersymmetry (SUSY) may allow us to relate the Classical Scale to the Quantum Scale, as well as Bosons to Fermions. This is all very relevant, but we must recognize that we live in an Observable Universe that seems to be finite both in spatial and temporal dimensions. Although 13+ billion years sounds like a relatively old age, it is still finite. We are limited from seeing beyond the Observable Universe by the Relativistic speed-of-light Scale limit. Thus, a physical infinity cannot exist within our Observable Universe – but rather, all very large numbers (such as Dirac's Large Number of $\sim 10^{40}$) and their inverses MUST be mandated by properties of Scales [**]. The author has proposed a Quantum Statistical Grand Unified Theory (QSGUT) [18] that may explain the origin and nature of these Scales. Likewise, we are limited from seeing within the Quantum Scale by the Planck Scale limit. Thus, Nottale [19] anticipates at least two more scales – one smaller than the Quantum Scale (the sub-quantum Dirac Sea Scale?), and one larger than the Observable Universe (the super-cosmic Multiverse Scale?). Scales are the result of phase transitions, and the phase transition (perhaps due to the breaking of the original TOE symmetry?) that caused Guth's Inflation [20] may have created more self-similar Scales [7] than we can see.

The expectation is that the Multiverse Scale has the greatest complexergy (complexity[††]-energy [19,21]), and large numbers approaching Infinity. Our Observable Universe is a self-similar scaled copy of the Multiverse with less complexergy and large numbers on the order of Dirac's Large Number or geometrical powers thereof. Quantum Gravity exists at the Multiverse Scale, but is separated from our scale by the speed-of-light scale limit. This scale limit causes a geometrical collapse into a buckyball-like lattice on the Multiverse side of the scale limit, and into a graphene-like lattice [22] on the Observable Universe side of the scale limit. These lattices provide the geometry for the Holographic Principle.

[**] Fine-tuning on the order of 10^{40} is not mathematically probable or philosophically reasonable.

[††] Shannon's Information Theory predicts complexity to scale as $N \times \ln(N)$, where N could have discrete properties, but ln(N) has continuous properties, and thus compensates for quasi-particle properties.

Self-similarity should apply to the Quantum Scale vs. the Dirac Sea Scale as well, and the Planck scale limit may also cause a geometrical collapse that prevents us from probing this sub-Quantum Scale and the origin of mass.

If a minimum of four different Scales exist, then "SUSY" must be more complex, and involve more hierarchal scales, than we normally expect. Not only might SUSY unite the Classical and Quantum Scales, but now we may require SUSY-like operators to intertwine the Quantum Scale with any smaller scales, and to intertwine the Observable Universe with any larger scales. This is fully consistent with Particle Physics expectations of fundamental particles with intrinsic spins of 0 (Higgs-boson-like), 1/2 (Fermions), 1 (Bosons), 3/2 (Gravitinos) and 2 (Gravitons), and the possibility that a set of "Hyper-SUSY" [18] operators may unite all of these spin characteristics and respective dominant Scales.

Conclusion

Nature is both Continuous and Discrete, and this feature necessitates (continuous) Wave – (discrete) Particle Dual behavior. The interrelationship between Scales and Supersymmetry (and Spin) may provide a mathematical umbrella capable of explaining these apparent paradoxes such as continuous vs. discrete, waves vs. particles, and the unnaturalness of "Infinity"; while simultaneously providing a framework for the ultimate unification of Bosons and Fermions. Although Physicists generally divide over String/ Membrane (continuous wave-like) and Kissing-Spheres/ CDT (discrete particle-like) frameworks, THEY MAY ALL BE HALF CORRECT!

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Appendix

Table A1 – Natural Logarithmic (ln = log $_e$) Properties of Golden Ratio (GR) Powers

l	ϕ^l GR Power	$\ln(\phi^l) = l \times \ln\phi$ GR Logarithm (ln)	ϕ^{-l} GR Inverse Power	$\ln(\phi^{-l}) = -l \times \ln\phi$ GR Inverse ln
0	1.0000000000 000...	+ 0.0000000000 000	1.0000000000 000...	– 0.0000000000 000
1	1.6180339887 499...	+ 0.4812118250 596	0.6180339887 499...	– 0.4812118250 596
2	2.6180339887 499...	+ 0.9624236501 192	0.3819660112 501...	– 0.9624236501 192
3	4.2360679774 998...	+ 1.4436354751 788	0.2360679774 998...	– 1.4436354751 788

Table A2 – Properties of Lucas Numbers and Golden Ratio (GR) Powers

l	n_l Lucas #	ϕ^l GR Power	$(-\phi)^{-l}$ GR Inverse Power
0	2	1.0000000000 000...	+ 1.0000000000 000...
1	1	1.6180339887 499...	– 0.6180339887 499...
2	3	2.6180339887 499...	+ 0.3819660112 501...
3	4	4.2360679774 998...	– 0.2360679774 998...
4	7	6.8541019662 497...	+ 0.1458980337 503...

Figure A1 – Nested Pentagram Chord Lengths

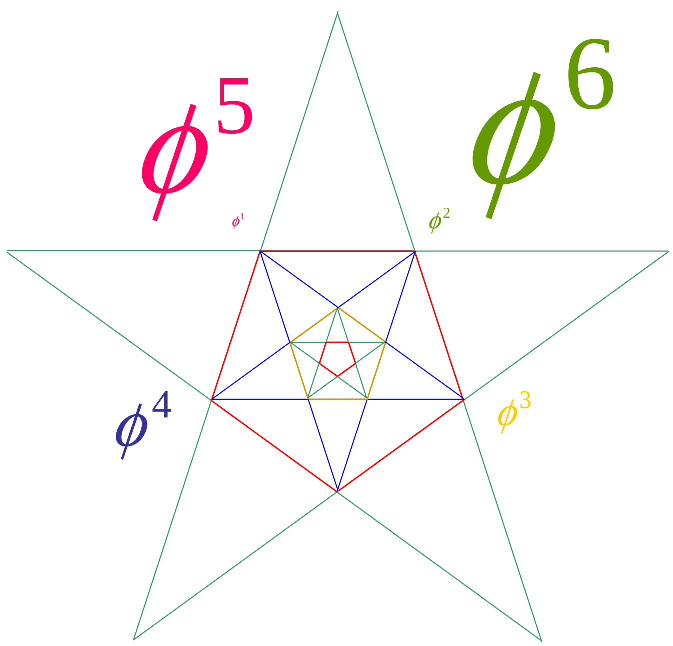

Figure idea courtesy of:
Scott Olsen, *The Golden Section: Nature's Greatest Secret*, Walker & Co., (2006).